A Drinfeld-type presentation of the orthosymplectic Yangians

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Abstract

We use the Gauss decomposition of the generator matrix in the R-matrix presentation of the Yangian for the orthosymplectic Lie superalgebra $\mathfrak{osp}_{N|2m}$ to produce its Drinfeld-type presentation. The results rely on the work of Gow (2007) concerning the $\mathfrak{gl}_{n|m}$ Yangians and our work with Jing and Liu (2018) devoted to the orthogonal and symplectic Yangians. One exception is the embedding theorem whose proof required a different approach in the super case.

1 Introduction

By the original definition of Drinfeld [6], the *Yangian* $\mathrm{Y}(\mathfrak{a})$ associated with a simple Lie algebra $\mathfrak{a}$ is a canonical deformation of the universal enveloping algebra $\mathrm{U}(\mathfrak{a}[u])$ in the class of Hopf algebras. The finite-dimensional irreducible representations of the algebra $\mathrm{Y}(\mathfrak{a})$ were classified in his subsequent work [7] with the use of a new presentation which is now often referred to as the *Drinfeld presentation*. It involves sufficiently many generators which are needed to identify the representations by their highest weights. It is well-known by Levendorskiĭ [24], that this presentation admits a reduced version involving a finite set of generators; see also [15] for its refined form and generalization to symmetrizable Kac–Moody Lie algebras.

The Drinfeld presentation is essential in the theory of *Yangian characters* or *q-characters* which were originally introduced by Knight [20] and by Frenkel and Reshetikhin [9] in the quantum affine algebra context. The theory was further developed in [8], [17] and [26], while an extensive review of the applications to integrable systems was given in [23]. The isomorphism between completions of the Yangians and quantum loop algebras constructed by Gautam and Toledano Laredo [10] also relies on the Drinfeld presentations.

The Yangian-type algebra associated with the general linear Lie algebra $\mathfrak{gl}_n$ was considered previously in the work of the Leningrad school on the quantum inverse scattering method, although this name for the algebra was not used; see e.g. review paper by Kulish and Sklyanin [22]. In this approach, the defining relations are written in the form of a single *RTT-relation* involving the *Yang R-matrix*; see also a brief discussion in [5, Sec. 7.5] explaining connections with integrable lattice models.

An explicit isomorphism between the R-matrix and Drinfeld presentations of the Yangian for $\mathfrak{gl}_n$ can be constructed with the use of the Gauss decomposition of the generator matrix $T(u)$, as was originally outlined in [7]; see [4] for a detailed proof. The same approach was used in

[18] to extend the isomorphisms to the remaining classical types B, C and D, while a different method to establish isomorphisms was developed in [16].

The Yangians associated with the general linear and orthosymplectic Lie superalgebras were first introduced in their R-matrix presentations. Nazarov defined the Yangian $\mathrm{Y}(\mathfrak{gl}_{n|m})$ in [27] by using a super-version of the Yang R-matrix, while the definition of the orthosymplectic Yangian $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ is due to Arnaudon *et al.* [1], where a super-version of the R-matrix originated in [31] was used. Presentations of the Yangian for $\mathfrak{sl}_{n|m}$ analogous to [7] and [24] were given by Stukopin [29], while the construction of [4] was extended to the Yangian $\mathrm{Y}(\mathfrak{gl}_{n|m})$ by Gow [13], where a Drinfeld-type presentation was devised together with an isomorphism with the R-matrix presentation. More general parabolic presentations of the Yangian $\mathrm{Y}(\mathfrak{gl}_{n|m})$, corresponding to arbitrary Borel subalgebras in $\mathfrak{gl}_{n|m}$ were given by Peng [28]; see also Tsymbaliuk [30].

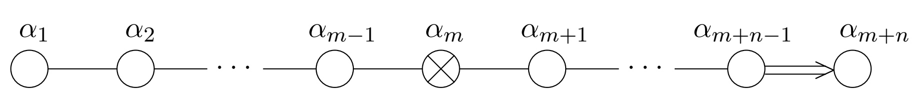
Our goal in this paper is to give a Drinfeld-type presentation for the orthosymplectic Yangians $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ with $N \geqslant 3$. To state the Main Theorem, introduce some notation related to the Lie superalgebras $\mathfrak{osp}_{N|2m}$, assuming that $m \geqslant 1$. If $N = 2n+1$ is odd, they form series B of simple Lie superalgebras, while in the case $N = 2n$ with $n \geqslant 2$ they belong to series D. We will consider both cases simultaneously whenever possible. We will assume that the simple roots of $\mathfrak{osp}_{N|2m}$ are $\alpha_1, \dots, \alpha_{m+n}$ with $\alpha_i = \varepsilon_i - \varepsilon_{i+1}$, for $i = 1, \dots, m+n-1$, and

$$\alpha_{m+n} = \begin{cases} \varepsilon_{m+n} & \text{for} \quad N = 2n+1, \\ \varepsilon_{m+n-1} + \varepsilon_{m+n} & \text{for} \quad N = 2n, \end{cases}$$

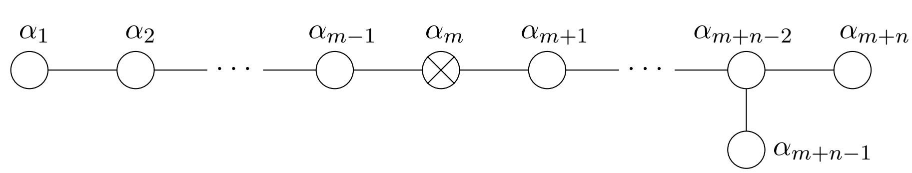
where $\varepsilon_1, \dots, \varepsilon_{m+n}$ is an orthogonal basis of a vector space with the bilinear form such that

$$(\varepsilon_i, \varepsilon_i) = \begin{cases} -1 & \text{for} \quad i = 1, \dots, m, \\ \ \ 1 & \text{for} \quad i = m+1, \dots, m+n. \end{cases}$$

This choice of simple roots corresponds to the standard Dynkin diagrams given by



for $\mathfrak{osp}_{2n+1|2m}$ with $n \geqslant 1$, and



for $\mathfrak{osp}_{2n|2m}$ with $n \geqslant 2$. In both cases, α_m is the only odd simple isotropic root. The associated Cartan matrix $C = [c_{ij}]_{i,j=1}^{m+n}$ is defined by $c_{ij} = (\alpha_i, \alpha_j)$ for series D, and by

$$c_{ij} = \begin{cases} (\alpha_i, \alpha_j) & \text{if} \quad i < m+n, \\ 2\,(\alpha_i, \alpha_j) & \text{if} \quad i = m+n, \end{cases}$$

for series B. Note that the $n \times n$ submatrix $[c_{ij}]_{i,j=m+1}^{m+n}$ coincides with the Cartan matrix associated with the simple Lie algebra of type D_n or B_n, respectively.

Main Theorem. *The Yangian* $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ *with* $N \geqslant 3$ *is isomorphic to the superalgebra with generators* κ_{ir}, ξ_{ir}^{+} *and* ξ_{ir}^{-}, *where* $i = 1, \dots, m+n$ *and* $r = 0, 1, \dots$. *The generators* $\xi_{mr}^{\pm}$ *are odd, while all the remaining generators are even. The defining relations have the form*

$$[\kappa_{i\,r}, \kappa_{j\,s}] = 0, \tag{1.1}$$

$$[\xi_{ir}^{+}, \xi_{j\,s}^{-}] = \delta_{ij}\, \kappa_{i\,r+s}, \tag{1.2}$$

$$[\kappa_{i\,0}, \xi_{j\,s}^{\pm}] = \pm\, (\alpha_i, \alpha_j)\, \xi_{j\,s}^{\pm}, \tag{1.3}$$

$$[\kappa_{i\,r+1}, \xi_{j\,s}^{\pm}] - [\kappa_{i\,r}, \xi_{j\,s+1}^{\pm}] = \pm\, \frac{(\alpha_i, \alpha_j)}{2} \Big(\kappa_{i\,r}\, \xi_{j\,s}^{\pm} + \xi_{j\,s}^{\pm}\, \kappa_{i\,r}\Big), \tag{1.4}$$

$$[\xi_{i\,r+1}^{\pm}, \xi_{j\,s}^{\pm}] - [\xi_{i\,r}^{\pm}, \xi_{j\,s+1}^{\pm}] = \pm\, \frac{(\alpha_i, \alpha_j)}{2} \Big(\xi_{i\,r}^{\pm}\, \xi_{j\,s}^{\pm} + \xi_{j\,s}^{\pm}\, \xi_{i\,r}^{\pm}\Big), \tag{1.5}$$

$$[\xi_{m\,r}^{\pm}, \xi_{m\,s}^{\pm}] = 0, \tag{1.6}$$

together with the Serre relations

$$\sum_{\sigma \in \mathfrak{S}_k} [\xi_{i r_{\sigma(1)}}^{\pm}, [\xi_{i r_{\sigma(2)}}^{\pm}, \dots [\xi_{i r_{\sigma(k)}}^{\pm}, \xi_{j\,s}^{\pm}] \dots]] = 0, \tag{1.7}$$

for $i \neq j$, *where we set* $k = 1 + |c_{ij}|$, *and the super Serre relations*

$$\Big[[\xi_{m-1\,r}^{\pm}, \xi_{m\,0}^{\pm}], [\xi_{m\,0}^{\pm}, \xi_{m+1\,s}^{\pm}]\Big] = 0. \tag{1.8}$$

In the formulation of the theorem we used square brackets to denote super-commutator

$$[a, b] = ab - ba\, (-1)^{p(a)p(b)}$$

for homogeneous elements a and b of parities $p(a)$ and $p(b)$. The subscripts take all possible admissible values. Note that relations (1.3) and (1.4) with $i = j = m$ imply

$$[\kappa_{m\,r}, \xi_{m\,s}^{\pm}] = 0. \tag{1.9}$$

Relation (1.5) for $i = j = m$ is implied by (1.6). If $m = 1$, then the super Serre relations (1.8) are omitted. By omitting relations (1.6) and (1.8), and taking $m = 0$, we recover the Drinfeld presentation of the Yangian $\mathrm{Y}(\mathfrak{o}_N)$ [7]; cf. [18].

A key role in the proof of the Main Theorem is played by the embedding theorem for the extended Yangians which shows that for any $m \geqslant 1$ the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m-2})$ can be regarded as a subalgebra of $\mathrm{X}(\mathfrak{osp}_{N|2m})$. Its counterpart for the orthogonal and symplectic Yangians was proved in our work with Jing and Liu [18, Thm 3.1]. However, that proof does not fully extend to the super case since the values of the R-matrix $R(1)$ used therein are not defined in general. Instead, we employ R-matrix calculations to produce a different argument.

As a next step, we follow [4], [13] and [18] to use the Gauss decomposition of the generator matrix $T(u)$ in the R-matrix presentation of the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m})$ to introduce the Gaussian generators. This leads to a Drinfeld-type presentation of the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m})$ (Theorem 6.1) which will then be used to derive the presentation of the Yangian $\mathrm{Y}(\mathfrak{osp}_{N|2m}) \subset \mathrm{X}(\mathfrak{osp}_{N|2m})$ given in the Main Theorem. As a part of the proof, we use a description of the center of the extended Yangian in terms of the Gaussian generators (Theorem 5.3). This is a super-counterpart of [18, Thm 5.8], although we use a different argument relying on Jacobi's ratio theorem for quasideterminants; see [11] and [21].

Drinfeld-type presentations of the Yangians $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ with $N = 1$ and $N = 2$ should require some modifications of the relations in the Main Theorem. Such a presentation of the Yangian $\mathrm{Y}(\mathfrak{osp}_{1|2})$ was given in [2]; it involves additional Serre relations of different kind. By the embedding theorem, the Drinfeld-type presentation of $\mathrm{Y}(\mathfrak{osp}_{2|2m})$ is largely determined by the case $m = 1$ which should rely on an isomorphism between the Yangians associated with the Lie superalgebras $\mathfrak{osp}_{2|2}$ and $\mathfrak{sl}_{1|2}$.

2 Basic properties of the orthosymplectic Yangian

Introduce the involution $i \mapsto i' = N + 2m - i + 1$ on the set $\{1, 2, \dots, N + 2m\}$. Consider the $\mathbb{Z}_2$-graded vector space $\mathbb{C}^{N|2m}$ over $\mathbb{C}$ with the canonical basis $e_1, e_2, \dots, e_{N+2m}$, where the vector e_i has the parity $\bar{\imath} \mod 2$ and

$$\bar{\imath} = \begin{cases} 1 & \text{for} \quad i = 1, \dots, m, m', \dots, 1', \\ 0 & \text{for} \quad i = m+1, \dots, (m+1)'. \end{cases}$$

The endomorphism algebra $\mathrm{End}\, \mathbb{C}^{N|2m}$ is then equipped with a $\mathbb{Z}_2$-gradation with the parity of the matrix unit e_{ij} found by $\bar{\imath} + \bar{\jmath} \mod 2$. We will identify the algebra of even matrices over a superalgebra $\mathcal{A}$ with the tensor product algebra $\mathrm{End}\, \mathbb{C}^{N|2m} \otimes \mathcal{A}$, so that a square matrix $A = [a_{ij}]$ of size $N + 2m$ is regarded as the element

$$A = \sum_{i,j=1}^{N+2m} e_{ij} \otimes a_{ij} (-1)^{\bar{\imath}\bar{\jmath} + \bar{\jmath}} \in \mathrm{End}\, \mathbb{C}^{N|2m} \otimes \mathcal{A},$$

where the entries a_{ij} are assumed to be homogeneous of parity $\bar{\imath} + \bar{\jmath} \mod 2$. The extra signs are necessary to keep the usual rule for the matrix multiplication. The involutive matrix *super-transposition* t is defined by $(A^t)_{ij} = a_{j'i'} (-1)^{\bar{\imath}\bar{\jmath} + \bar{\jmath}} \theta_i \theta_j$, where we set

$$\theta_i = \begin{cases} 1 & \text{for} \quad i = 1, \dots, N + m, \\ -1 & \text{for} \quad i = N + m + 1, \dots, N + 2m. \end{cases}$$

This super-transposition is associated with the bilinear form on the space $\mathbb{C}^{N|2m}$ defined by the anti-diagonal matrix $G = [g_{ij}]$ with $g_{ij} = \delta_{ij'} \theta_i$.

A standard basis of the general linear Lie superalgebra $\mathfrak{gl}_{N|2m}$ is formed by elements E_{ij} of the parity $\bar{\imath}+\bar{\jmath} \mod 2$ for $1 \leqslant i,j \leqslant N+2m$ with the commutation relations

$$[E_{ij}, E_{kl}] = \delta_{kj}\, E_{il} - \delta_{il}\, E_{kj}(-1)^{(\bar{\imath}+\bar{\jmath})(\bar{k}+\bar{l})}.$$

We will regard the orthosymplectic Lie superalgebra $\mathfrak{osp}_{N|2m}$ associated with the bilinear from defined by G as the subalgebra of $\mathfrak{gl}_{N|2m}$ spanned by the elements

$$F_{ij} = E_{ij} - E_{j'i'}(-1)^{\bar{\imath}\bar{\jmath}+\bar{\imath}}\,\theta_i\theta_j.$$

Introduce the permutation operator P by

$$P = \sum_{i,j=1}^{N+2m} e_{ij} \otimes e_{ji}(-1)^{\bar{\jmath}} \in \operatorname{End}\mathbb{C}^{N|2m} \otimes \operatorname{End}\mathbb{C}^{N|2m} \tag{2.1}$$

and set

$$Q = \sum_{i,j=1}^{N+2m} e_{ij} \otimes e_{i'j'}(-1)^{\bar{\imath}\bar{\jmath}}\,\theta_i\theta_j \in \operatorname{End}\mathbb{C}^{N|2m} \otimes \operatorname{End}\mathbb{C}^{N|2m}. \tag{2.2}$$

The R-*matrix* associated with $\mathfrak{osp}_{N|2m}$ is the rational function in u given by

$$R(u) = 1 - \frac{P}{u} + \frac{Q}{u-\kappa}, \qquad \kappa = \frac{N}{2} - m - 1.$$

This is a super-version of the R-matrix originally found in [31]. Following [1], we define the *extended Yangian* $\mathrm{X}(\mathfrak{osp}_{N|2m})$ as a $\mathbb{Z}_2$-graded algebra with generators $t_{ij}^{(r)}$ of parity $\bar{\imath}+\bar{\jmath} \mod 2$, where $1 \leqslant i,j \leqslant N+2m$ and $r = 1,2,\dots$, satisfying the following defining relations. Introduce the formal series

$$t_{ij}(u) = \delta_{ij} + \sum_{r=1}^{\infty} t_{ij}^{(r)}\, u^{-r} \in \mathrm{X}(\mathfrak{osp}_{N|2m})[[u^{-1}]] \tag{2.3}$$

and combine them into the matrix $T(u) = [t_{ij}(u)]$. Consider the elements of the tensor product algebra $\operatorname{End}\mathbb{C}^{N|2m} \otimes \operatorname{End}\mathbb{C}^{N|2m} \otimes \mathrm{X}(\mathfrak{osp}_{N|2m})[[u^{-1}]]$ given by

$$T_1(u) = \sum_{i,j=1}^{N+2m} e_{ij} \otimes 1 \otimes t_{ij}(u)(-1)^{\bar{\imath}\bar{\jmath}+\bar{\jmath}} \quad \text{and} \quad T_2(u) = \sum_{i,j=1}^{N+2m} 1 \otimes e_{ij} \otimes t_{ij}(u)(-1)^{\bar{\imath}\bar{\jmath}+\bar{\jmath}}.$$

The defining relations for the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$ take the form of the RTT-*relation*

$$R(u-v)\, T_1(u)\, T_2(v) = T_2(v)\, T_1(u)\, R(u-v). \tag{2.4}$$

For $m = 0$ the algebra $\mathrm{X}(\mathfrak{osp}_{N|0})$ coincides with the extended Yangian $\mathrm{X}(\mathfrak{o}_N)$ associated with the orthogonal Lie algebra $\mathfrak{o}_N$.

As shown in [1], the product $T(u-\kappa)\, T^t(u)$ is a scalar matrix with

$$T(u-\kappa)\, T^t(u) = c(u)1, \tag{2.5}$$

where $c(u)$ is a series in u^{-1}. All its coefficients belong to the center $\mathrm{ZX}(\mathfrak{osp}_{N|2m})$ of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ and freely generate the center; cf. the Lie algebra case considered in [3].

The *Yangian* $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ is defined as the subalgebra of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ which consists of the elements stable under the automorphisms

$$t_{ij}(u) \mapsto f(u)\, t_{ij}(u) \tag{2.6}$$

for all series $f(u) \in 1 + u^{-1}\mathbb{C}[[u^{-1}]]$. As in the non-super case [3], we have the tensor product decomposition

$$\mathrm{X}(\mathfrak{osp}_{N|2m}) = \mathrm{ZX}(\mathfrak{osp}_{N|2m}) \otimes \mathrm{Y}(\mathfrak{osp}_{N|2m}); \tag{2.7}$$

see also [14]. The Yangian $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ is isomorphic to the quotient of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ by the relation $c(u) = 1$. In the case $m = 0$ the Yangian $\mathrm{Y}(\mathfrak{osp}_{N|0})$ coincides with $\mathrm{Y}(\mathfrak{o}_N)$.

An explicit form of the defining relations (2.4) can be written in terms of the series (2.3) as follows:

$$\begin{aligned}\big[t_{ij}(u), t_{kl}(v)\big] &= \frac{1}{u-v}\Big(t_{kj}(u)\, t_{il}(v) - t_{kj}(v)\, t_{il}(u)\Big)(-1)^{\bar\imath\bar\jmath+\bar\imath\bar k+\bar\jmath\bar k}\\ &- \frac{1}{u-v-\kappa}\Big(\delta_{ki'}\sum_{p=1}^{N+2m} t_{pj}(u)\, t_{p'l}(v)(-1)^{\bar\imath+\bar\imath\bar\jmath+\bar\jmath\bar p}\,\theta_i\theta_p\\ &\qquad - \delta_{lj'}\sum_{p=1}^{N+2m} t_{kp'}(v)\, t_{ip}(u)(-1)^{\bar\imath\bar k+\bar\jmath\bar k+\bar\imath\bar p}\,\theta_{j'}\theta_{p'}\Big).\end{aligned} \tag{2.8}$$

The mapping

$$t_{ij}(u) \mapsto t_{ij}(u+a), \qquad a \in \mathbb{C},$$

defines an automorphism of $\mathrm{X}(\mathfrak{osp}_{N|2m})$, while the mapping

$$\tau : t_{ij}(u) \mapsto t_{ji}(u)(-1)^{\bar\imath\bar\jmath+\bar\jmath} \tag{2.9}$$

defines an anti-automorphism. The latter property is understood in the sense that

$$\tau(ab) = \tau(b)\tau(a)(-1)^{p(a)p(b)}$$

for homogeneous elements a and b of the Yangian.

The universal enveloping algebra $\mathrm{U}(\mathfrak{osp}_{N|2m})$ can be regarded as a subalgebra of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ via the embedding

$$F_{ij} \mapsto \frac{1}{2}\Big(t_{ij}^{(1)} - t_{j'i'}^{(1)}(-1)^{\bar\jmath+\bar\imath\bar\jmath}\,\theta_i\theta_j\Big)(-1)^{\bar\imath}. \tag{2.10}$$

This fact relies on the Poincaré–Birkhoff–Witt theorem for the orthosymplectic Yangian which was pointed out in [1] and a detailed proof is given in [14]; cf. [3, Sec. 3]. It states that the associated graded algebra for $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ is isomorphic to $\mathrm{U}(\mathfrak{osp}_{N|2m}[u])$. The algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$ is generated by the coefficients of the series $c(u)$ and $t_{ij}(u)$ with the conditions

$$\begin{aligned} &i + j \leqslant N + 2m + 1 \qquad \text{for} \quad i = 1, \dots, m, m', \dots, 1' \quad \text{and}\\ &i + j < N + 2m + 1 \qquad \text{for} \quad i = m+1, \dots, (m+1)'.\end{aligned}$$

Moreover, given any total ordering on the set of the generators, the ordered monomials with the powers of odd generators not exceeding 1, form a basis of the algebra.

3 The embedding theorem

Let $A = [a_{ij}]$ be a $p \times p$ matrix over a ring with 1. Denote by A^{ij} the matrix obtained from A by deleting the i-th row and j-th column. Suppose that the matrix A^{ij} is invertible. The *ij-th quasideterminant of A* is defined by the formula

$$|A|_{ij} = a_{ij} - r_i^j (A^{ij})^{-1} c_j^i, \tag{3.1}$$

where r_i^j is the row matrix obtained from the i-th row of A by deleting the element a_{ij}, and c_j^i is the column matrix obtained from the j-th column of A by deleting the element a_{ij}; see [11]. The quasideterminant $|A|_{ij}$ is also denoted by boxing the entry a_{ij},

$$|A|_{ij} = \begin{vmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1p} \\ & \dots & & \dots & \\ a_{i1} & \dots & \boxed{a_{ij}} & \dots & a_{ip} \\ & \dots & & \dots & \\ a_{p1} & \dots & a_{pj} & \dots & a_{pp} \end{vmatrix}.$$

If the matrix A is invertible and the (j, i) entry of A^{-1} is invertible, then the quasideterminant can be found by

$$|A|_{ij} = \big((A^{-1})_{ji}\big)^{-1}. \tag{3.2}$$

Suppose that $(\{i\}, L, U)$ and $(\{j\}, M, V)$ are partitions of the set $\{1, 2, \dots, p\}$ such that $|L| = |M|$ and $|U| = |V|$. Then, according to Jacobi's ratio theorem for quasideterminants,

$$\big|A_{U\cup\{i\},V\cup\{j\}}\big|_{ij}^{-1} = \big|B_{M\cup\{j\},L\cup\{i\}}\big|_{ji}, \tag{3.3}$$

where $B = A^{-1}$ and C_{PQ} denotes the submatrix of a matrix C whose rows are labelled by the set P and columns labelled by a set Q; see [11] and [21].

Now consider the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m-2})$ with $m \geqslant 1$ and let the indices of the generators $t_{ij}^{(r)}$ range over the sets $2 \leqslant i, j \leqslant 2'$ and $r = 1, 2, \dots$. The following is a super-version of [18, Thm 3.1].

Theorem 3.1. *For $m \geqslant 1$ the mapping*

$$t_{ij}(u) \mapsto \begin{vmatrix} t_{11}(u) & t_{1j}(u) \\ t_{i1}(u) & \boxed{t_{ij}(u)} \end{vmatrix}, \qquad 2 \leqslant i, j \leqslant 2', \tag{3.4}$$

defines an injective homomorphism $\mathrm{X}(\mathfrak{osp}_{N|2m-2}) \to \mathrm{X}(\mathfrak{osp}_{N|2m})$. Moreover, its restriction to the subalgebra $\mathrm{Y}(\mathfrak{osp}_{N|2m-2})$ defines an injective homomorphism $\mathrm{Y}(\mathfrak{osp}_{N|2m-2}) \to \mathrm{Y}(\mathfrak{osp}_{N|2m})$.

Proof. The following elements of the algebra $\mathrm{End}\,\mathbb{C}^{N|2m}$ will be used throughout the proof:

$$I = \sum_{i=2}^{2'} e_{ii} \qquad \text{and} \qquad J = \sum_{i=1}^{2'} e_{ii}.$$

For any $A \in \operatorname{End}\mathbb{C}^{N|2m}$ the product IAI will be understood as an element of $\operatorname{End}\mathbb{C}^{N|2m-2}$, where we identify $\operatorname{End}\mathbb{C}^{N|2m-2}$ with the subalgebra of $\operatorname{End}\mathbb{C}^{N|2m}$ spanned by the basis elements e_{ij} with $2 \leqslant i, j \leqslant 2'$. Introduce the matrix

$$G(u) = I\overline{T}(u)^{-1}I,$$

where $\overline{T}(u) = JT(u)J$, and we regard $\overline{T}(u)$ as the submatrix of $T(u)$ obtained by deleting the last row and column. Write this submatrix in the block form

$$\overline{T}(u) = \begin{bmatrix} A(u) & B(u) \\ C(u) & D(u) \end{bmatrix}$$

according to the partition $\{1\} \cup \{2, \dots, 2'\}$ of the row and column numbers, so that, in particular, $A(u) = t_{11}(u)$. Then using the block multiplication of matrices, we find that

$$G(u) = \Big(D(u) - C(u)A(u)^{-1}B(u)\Big)^{-1};$$

see e.g. [25, Lemma 1.11.1]. Therefore, the (i, j) entry of the matrix $G(u)^{-1}$ coincides with the series given by the quasideterminant

$$t_{ij}(u) - t_{i1}(u)t_{11}(u)^{-1}t_{1j}(u) \tag{3.5}$$

appearing in (3.4). This means that in order to show that the map (3.4) defines a homomorphism, it will be enough to verify that $G(u)$ satisfies the relation

$$\overline{R}(u-v)\, G_2(v)\, G_1(u) = G_1(u)\, G_2(v)\, \overline{R}(u-v), \tag{3.6}$$

where $G_1(u)$ and $G_2(u)$ are elements of the tensor product algebra

$$\operatorname{End}\mathbb{C}^{N|2m-2} \otimes \operatorname{End}\mathbb{C}^{N|2m-2} \otimes \mathrm{X}(\mathfrak{osp}_{N|2m})[[u^{-1}]]$$

with the copies of the endomorphism algebra respectively labelled by 1 and 2 as in (2.4). Here we use the R-matrix

$$\overline{R}(u) = 1 - \frac{\overline{P}}{u} + \frac{\overline{Q}}{u - \kappa - 1}$$

associated with $\mathrm{X}(\mathfrak{osp}_{N|2m-2})$ so that the operators $\overline{P}$ and $\overline{Q}$ are given by the respective formulas (2.1) and (2.2) with the summations restricted to the set $i, j \in \{2, \dots, 2'\}$.

Introduce elements of the algebra $\operatorname{End}\mathbb{C}^{N|2m} \otimes \operatorname{End}\mathbb{C}^{N|2m}$ by

$$K = K^+ + K^- \qquad \text{and} \qquad \check{K} = \check{K}^+ + \check{K}^-,$$

where

$$K^+ = \sum_{i=2}^{2'} e_{i1} \otimes e_{i'1'}(-1)^{\bar{\imath}}\,\theta_i, \qquad \check{K}^+ = \sum_{i=2}^{2'} e_{1i} \otimes e_{1'i'}(-1)^{\bar{\imath}}\,\theta_i,$$

and

$$K^{-}=-\sum_{i=2}^{2'} e_{i1'}\otimes e_{i'1}(-1)^{\bar{\imath}}\,\theta_i,\qquad \check{K}^{-}=-\sum_{i=2}^{2'} e_{1'i}\otimes e_{1i'}(-1)^{\bar{\imath}}\,\theta_i.$$

Note the relations

$$J_1J_2Q=\overline{Q}+K\qquad\text{and}\qquad Q\,J_1J_2=\overline{Q}+\check{K}, \tag{3.7}$$

where the subscripts indicate the element J taken in the respective copy of the endomorphism algebra with the identity component in the other copy.

Multiply both sides of (2.4) by J_1J_2 from the left and from the right to get the relation

$$\begin{aligned}\widetilde{R}(u-v)\,\overline{T}_1(u)\,\overline{T}_2(v)-\overline{T}_2(v)\,\overline{T}_1(u)\,\widetilde{R}(u-v)&\\ =-\frac{1}{u-v-\kappa}\Big(K\,T_1(u)\,T_2(v)J_1J_2-J_1J_2T_2(v)\,T_1(u)\,\check{K}\Big),&\end{aligned} \tag{3.8}$$

where we set

$$\widetilde{R}(u)=1-\frac{\widetilde{P}}{u}+\frac{\overline{Q}}{u-\kappa},\qquad \widetilde{P}=\sum_{i,j=1}^{2'} e_{ij}\otimes e_{ji}(-1)^{\bar{\jmath}}.$$

As a next step, multiply both sides of (2.4) by K^{+} from the left. Since $K^{+}P=K^{-}$ and $K^{+}Q=-\overline{Q}-K$, we get

$$\Big(K^{+}-\frac{K^{-}}{u-v}-\frac{\overline{Q}+K}{u-v-\kappa}\Big)\,T_1(u)\,T_2(v)=K^{+}T_2(v)\,T_1(u)\,R(u-v)$$

and hence

$$\begin{aligned}K\,T_1(u)\,T_2(v)=\frac{1}{u-v-\kappa-1}\,\overline{Q}\,T_1(u)\,T_2(v)&+\frac{(u-v+1)(u-v-\kappa)}{(u-v)(u-v-\kappa-1)}\,K^{-}T_1(u)\,T_2(v)\\ &+\frac{u-v-\kappa}{u-v-\kappa-1}\,K^{+}T_2(v)\,T_1(u)\,R(u-v).\end{aligned}$$

Performing a similar calculation after multiplying both sides of (2.4) by $\check{K}^{+}$ from the right, and then rearranging (3.8), we come to the relation

$$\begin{aligned}\Big(1-\frac{\widetilde{P}}{u-v}+\frac{\overline{Q}}{u-v-\kappa-1}\Big)\,\overline{T}_1(u)\,\overline{T}_2(v)-\overline{T}_2(v)\,\overline{T}_1(u)\,\Big(1-\frac{\widetilde{P}}{u-v}+\frac{\overline{Q}}{u-v-\kappa-1}\Big)&\\ =-\frac{u-v+1}{(u-v)(u-v-\kappa-1)}\Big(K^{-}T_1(u)\,T_2(v)J_1J_2-J_1J_2\,T_2(v)\,T_1(u)\,\check{K}^{-}\Big)&\\ -\frac{1}{u-v-\kappa-1}\Big(K^{+}T_2(v)\,T_1(u)\,R(u-v)\,J_1J_2-J_1J_2\,R(u-v)\,T_1(u)\,T_2(v)\,\check{K}^{+}\Big).&\end{aligned}$$

Now transform this relation by multiplying its both sides from the left and from the right first by $\overline{T}_2(v)^{-1}$, then by $\overline{T}_1(u)^{-1}$, and then by I_1I_2. After this transformation, some terms on the right hand side will vanish. Namely,

$$K^{-}T_1(u)\,T_2(v)J_1J_2\,\overline{T}_2(v)^{-1}\overline{T}_1(u)^{-1}I_1I_2=0,$$

which follows by writing

$$T_2(v)\, J_2\, \overline{T}_2(v)^{-1} = J_2 + (1 \otimes e_{1'1'})\, T_2(v)\, J_2\, \overline{T}_2(v)^{-1},$$

and observing that $K^-(1 \otimes e_{1'1'}) = 0$ and

$$K^- T_1(u)\, J_1 J_2\, \overline{T}_1(u)^{-1} I_1 I_2 = K^- I_2\, T_1(u)\, J_1\, \overline{T}_1(u)^{-1} I_1 = 0.$$

Similarly,

$$I_1 I_2 \overline{T}_1(u)^{-1} \overline{T}_2(v)^{-1} J_1 J_2\, T_2(v)\, T_1(u)\, \check{K}^- = 0.$$

Therefore, taking into account the identities $K^+ = I_1 I_2\, K^+$ and $\check{K}^- = \check{K}^- I_1 I_2$, as a result of the transformation, we find that the difference

$$\overline{R}(u-v)\, G_2(v)\, G_1(u) - G_1(u)\, G_2(v)\, \overline{R}(u-v) \tag{3.9}$$

equals

$$\begin{aligned} \frac{1}{u-v-\kappa-1} \Big(& G_1(u)\, G_2(v)\, K^+ T_2(v)\, T_1(u)\, R(u-v)\, J_1 J_2\, \overline{T}_2(v)^{-1} \overline{T}_1(u)^{-1} I_1 I_2 \\ & - I_1 I_2 \overline{T}_1(u)^{-1} \overline{T}_2(v)^{-1} J_1 J_2\, R(u-v)\, T_1(u)\, T_2(v)\, \check{K}^+\, G_2(v)\, G_1(u) \Big). \end{aligned} \tag{3.10}$$

To transform the first product, use the second relation in (3.7) to write

$$R(u-v)\, J_1 J_2 = J_1 J_2\, \widetilde{R}(u-v) + \frac{1}{u-\kappa}\, \check{K}.$$

Furthermore, due to (3.8), the difference

$$\overline{T}_1(u)^{-1} \overline{T}_2(v)^{-1}\, \widetilde{R}(u-v) - \widetilde{R}(u-v)\, \overline{T}_2(v)^{-1} \overline{T}_1(u)^{-1}$$

equals the right hand side of (3.8) multiplied from the left and from the right by $\overline{T}_2(v)^{-1}$ and then by $\overline{T}_1(u)^{-1}$. Hence, replacing $\widetilde{R}(u-v)\, \overline{T}_2(v)^{-1} \overline{T}_1(u)^{-1}$ by the resulting expression, we can bring the first product in (3.10) to the form

$$\begin{aligned} & G_1(u)\, G_2(v)\, K^+ T_2(v)\, J_2\, T_1(u) J_1\, \overline{T}_1(u)^{-1} \overline{T}_2(v)^{-1} \widetilde{R}(u-v) I_1 I_2 \\ + \frac{1}{u-v-\kappa}\, & G_1(u)\, G_2(v)\, K^+ T_2(v)\, J_2\, T_1(u) J_1\, \overline{T}_1(u)^{-1} \overline{T}_2(v)^{-1} \\ & \times \Big(K\, T_1(u)\, T_2(v) J_1 J_2 - J_1 J_2 T_2(v)\, T_1(u)\, \check{K} \Big)\, \overline{T}_2(v)^{-1} \overline{T}_1(u)^{-1}\, I_1 I_2 \\ & + \frac{1}{u-v-\kappa}\, G_1(u)\, G_2(v)\, K^+ T_2(v) T_1(u) \check{K}\, G_2(v)\, G_1(u). \end{aligned}$$

Write

$$T_1(u) J_1\, \overline{T}_1(u)^{-1} = J_1 + (e_{1'1'} \otimes 1) T_1(u) J_1\, \overline{T}_1(u)^{-1} \tag{3.11}$$

to see that two products in the expression vanish. Indeed, since $K^+(e_{1'1'}\otimes 1)=0$, and $K=I_1K$, we get

$$K^+T_2(v)\,J_2\,J_1\overline{T}_2(v)^{-1}\widetilde{R}(u-v)I_1I_2=K^+I_1\,T_2(v)\,J_2\,\overline{T}_2(v)^{-1}I_2\widetilde{R}(u-v)=0$$

and

$$K^+T_2(v)\,J_2\,J_1\overline{T}_2(v)^{-1}K=K^+I_1\,T_2(v)\,J_2\,\overline{T}_2(v)^{-1}K=0.$$

Hence, using (3.11) again to simplify the remaining terms, we find that first product in (3.10) takes the form

$$-\frac{1}{u-v-\kappa}\,G_1(u)\,G_2(v)\,K^+T_2(v)J_2\,\overline{T}_2(v)^{-1}J_2\,T_2(v)\,T_1(u)\check{K}\,G_2(v)\,G_1(u)$$
$$+\frac{1}{u-v-\kappa}\,G_1(u)\,G_2(v)\,K^+T_2(v)T_1(u)\check{K}\,G_2(v)\,G_1(u). \quad (3.12)$$

Since

$$\overline{T}_2(v)^{-1}J_2\,T_2(v)=J_2+\overline{T}_2(v)^{-1}J_2\,T_2(v)(1\otimes e_{1'1'})$$

and $(1\otimes e_{1'1'})\check{K}^-=0$, both occurrences of $\check{K}$ in (3.12) can be replaced by $\check{K}^+$, because the components with $\check{K}^-$ cancel.

Performing a similar calculation for the second product in (3.10), we can conclude that the difference (3.9) equals

$$-\frac{1}{(u-v-\kappa)(u-v-\kappa-1)}\,G_1(u)\,G_2(v)\,W\,G_2(v)\,G_1(u)$$

with

$$W=K^+\Big(T_1(u)\,T_2(v)-T_2(v)\,T_1(u)$$
$$-\,T_1(u)\,T_2(v)J_2\,\overline{T}_2(v)^{-1}J_2\,T_2(v)+T_2(v)J_2\,\overline{T}_2(v)^{-1}J_2\,T_2(v)\,T_1(u)\Big)\check{K}^+.$$

The expression W can be written as

$$W=K^+\Big[t_{11}(u),h_{1'}(v)\Big]\check{K}^+,$$

where $h_{1'}(v)$ denotes the $(1',1')$ entry of the matrix

$$T(v)-T(v)J\,\overline{T}(v)^{-1}J\,T(v).$$

According to (3.1), the series $h_{1'}(v)$ coincides with the quasideterminant $|T(v)|_{1'1'}$. On the other hand, (2.5) implies

$$T(v)^{-1}=c(v+\kappa)^{-1}\,T^{\,t}(v+\kappa).$$

Hence, (3.2) yields

$$h_{1'}(v)=c(v+\kappa)\,t_{11}(v+\kappa)^{-1}. \quad (3.13)$$

By the defining relations (2.8), we have $[t_{11}(u), t_{11}(v)] = 0$, and since the coefficients of the series $c(v)$ belong to the center of the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$, we get $W = 0$ thus proving (3.6).

The remaining parts of the theorem are verified by the same argument as for its non-super counterpart [18, Thm. 3.1]. Namely, to verify that the homomorphism (3.4) is injective, we pass to the associated graded algebras, where the ascending filtrations on the extended Yangians are defined by setting $\deg t_{ij}^{(r)} = r - 1$ for all $r \geqslant 1$. The injectivity of the homomorphism $\mathrm{gr}\, \mathrm{X}(\mathfrak{osp}_{N|2m-2}) \to \mathrm{gr}\, \mathrm{X}(\mathfrak{osp}_{N|2m})$ of the associated graded algebras follows from the Poincaré–Birkhoff–Witt theorem for $\mathrm{X}(\mathfrak{osp}_{N|2m-2})$.

The homomorphism (3.4) commutes with the automorphism μ_f defined in (2.6) associated with an arbitrary series $f(u)$. Therefore, the image of the restriction of this homomorphism to the Yangian $\mathrm{Y}(\mathfrak{osp}_{N|2m-2})$ is contained in the subalgebra $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ of $\mathrm{X}(\mathfrak{osp}_{N|2m})$. Hence this restriction defines an injective homomorphism $\mathrm{Y}(\mathfrak{osp}_{N|2m-2}) \to \mathrm{Y}(\mathfrak{osp}_{N|2m})$. □

We point out some consequences of Theorem 3.1 which are verified in the same way as in the non-super case; see [18, Sec. 3].

Suppose that $\ell \leqslant m + n$ for type B and $\ell \leqslant m + n - 1$ for type D.

Corollary 3.2. *The mapping*

$$\psi_\ell : t_{ij}(u) \mapsto \begin{vmatrix} t_{11}(u) & \dots & t_{1\ell}(u) & t_{1j}(u) \\ \dots & \dots & \dots & \dots \\ t_{\ell 1}(u) & \dots & t_{\ell\ell}(u) & t_{\ell j}(u) \\ t_{i1}(u) & \dots & t_{i\ell}(u) & \boxed{t_{ij}(u)} \end{vmatrix}, \qquad \ell + 1 \leqslant i, j \leqslant (\ell + 1)', \tag{3.14}$$

defines an injective homomorphism

$$\mathrm{X}(\mathfrak{osp}_{N|2m-2\ell}) \to \mathrm{X}(\mathfrak{osp}_{N|2m}), \qquad \text{if} \quad \ell < m,$$

and an injective homomorphism

$$\mathrm{X}(\mathfrak{o}_{N+2m-2\ell}) \to \mathrm{X}(\mathfrak{osp}_{N|2m}), \qquad \text{if} \quad \ell \geqslant m,$$

where the generators $t_{ij}^{(r)}$ of the respective extended Yangians $\mathrm{X}(\mathfrak{osp}_{N|2m-2\ell})$ and $\mathrm{X}(\mathfrak{o}_{N+2m-2\ell})$ are labelled by the indices $\ell + 1 \leqslant i, j \leqslant (\ell + 1)'$.

Moreover, the restriction of the map to the Yangian defines injective homomorphisms

$$\mathrm{Y}(\mathfrak{osp}_{N|2m-2\ell}) \to \mathrm{Y}(\mathfrak{osp}_{N|2m}), \qquad \text{if} \quad \ell < m,$$

and

$$\mathrm{Y}(\mathfrak{o}_{N+2m-2\ell}) \to \mathrm{Y}(\mathfrak{osp}_{N|2m}), \qquad \text{if} \quad \ell \geqslant m.$$

The embeddings (3.14) possess the following consistency property; cf. [4]. We will write $\psi_\ell^{(N+2m)}$ for the embedding map ψ_ℓ in Corollary 3.2. Then we have the equality of maps

$$\psi_k^{(N+2m)} \circ \psi_\ell^{(N+2m-2k)} = \psi_{k+\ell}^{(N+2m)}. \tag{3.15}$$

Corollary 3.3. *For any $1 \leqslant a, b \leqslant \ell$, the coefficients of the series $t_{ab}(u)$ commute with the coefficients of the quasideterminants in (3.14) for all $\ell + 1 \leqslant i, j \leqslant (\ell + 1)'$ in the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$.* □

4 Gaussian generators

Apply the Gauss decomposition to the generator matrix $T(u)$ associated with the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m})$,

$$T(u) = F(u)\, H(u)\, E(u), \tag{4.1}$$

where $F(u)$, $H(u)$ and $E(u)$ are uniquely determined matrices of the form

$$F(u) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ f_{21}(u) & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f_{1'1}(u) & f_{1'2}(u) & \dots & 1 \end{bmatrix}, \qquad E(u) = \begin{bmatrix} 1 & e_{12}(u) & \dots & e_{11'}(u) \\ 0 & 1 & \dots & e_{21'}(u) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix},$$

and $H(u) = \mathrm{diag}\,\big[h_1(u), \dots, h_{1'}(u)\big]$. Recall the well-known formulas for the entries of the matrices $F(u)$, $H(u)$ and $E(u)$ in terms of quasideterminants [12]; see also [25, Sec. 1.11]. We have

$$h_i(u) = \begin{vmatrix} t_{1\,1}(u) & \dots & t_{1\,i-1}(u) & t_{1\,i}(u) \\ \vdots & \ddots & \vdots & \vdots \\ t_{i-1\,1}(u) & \dots & t_{i-1\,i-1}(u) & t_{i-1\,i}(u) \\ t_{i\,1}(u) & \dots & t_{i\,i-1}(u) & \boxed{t_{i\,i}(u)} \end{vmatrix}, \qquad i = 1, \dots, 1', \tag{4.2}$$

whereas

$$e_{ij}(u) = h_i(u)^{-1} \begin{vmatrix} t_{1\,1}(u) & \dots & t_{1\,i-1}(u) & t_{1\,j}(u) \\ \vdots & \ddots & \vdots & \vdots \\ t_{i-1\,1}(u) & \dots & t_{i-1\,i-1}(u) & t_{i-1\,j}(u) \\ t_{i\,1}(u) & \dots & t_{i\,i-1}(u) & \boxed{t_{i\,j}(u)} \end{vmatrix} \tag{4.3}$$

and

$$f_{ji}(u) = \begin{vmatrix} t_{1\,1}(u) & \dots & t_{1\,i-1}(u) & t_{1\,i}(u) \\ \vdots & \ddots & \vdots & \vdots \\ t_{i-1\,1}(u) & \dots & t_{i-1\,i-1}(u) & t_{i-1\,i}(u) \\ t_{j\,1}(u) & \dots & t_{j\,i-1}(u) & \boxed{t_{j\,i}(u)} \end{vmatrix} h_i(u)^{-1} \tag{4.4}$$

for $1 \leqslant i < j \leqslant 1'$.

We will need the formulas for the action of the anti-automorphism τ of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ defined in (2.9) on the Gaussian generators.

Lemma 4.1. *Under the anti-automorphism τ we have*

$$\tau : e_{ij}(u) \mapsto f_{ji}(u)(-1)^{\bar{\imath}\bar{\jmath}+\bar{\jmath}}, \qquad f_{ji}(u) \mapsto e_{ij}(u)(-1)^{\bar{\imath}\bar{\jmath}+\bar{\imath}}, \tag{4.5}$$

for $i < j$, and $\tau : h_i(u) \mapsto h_i(u)$ for all i.

Proof. We have the following relations for the matrix entries implied by (4.1):

$$t_{ii}(u) = h_i(u) + \sum_{k=1}^{i-1} f_{ik}(u)\, h_k(u)\, e_{ki}(u)$$

for $i = 1, \dots, 1'$, and

$$t_{ij}(u) = h_i(u)\, e_{ij}(u) + \sum_{k=1}^{i-1} f_{ik}(u)\, h_k(u)\, e_{kj}(u),$$
$$t_{ji}(u) = f_{ji}(u)\, h_i(u) + \sum_{k=1}^{i-1} f_{jk}(u)\, h_k(u)\, e_{ki}(u),$$

for $i < j$. The required formulas follow by applying τ to both sides of the relations and using the induction on i. □

Assuming that $\ell \leqslant m+n$ for type B and $\ell \leqslant m+n-1$ for type D, use the superscript $[\ell]$ to indicate square submatrices corresponding to rows and columns labelled by $\ell+1, \dots, (\ell+1)'$. In particular,

$$F^{[\ell]}(u) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ f_{\ell+2\,\ell+1}(u) & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ f_{(\ell+1)'\,\ell+1}(u) & \dots & f_{(\ell+1)'\,(\ell+2)'}(u) & 1 \end{bmatrix},$$

$$E^{[\ell]}(u) = \begin{bmatrix} 1 & e_{\ell+1\,\ell+2}(u) & \dots & e_{\ell+1\,(\ell+1)'}(u) \\ 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & e_{(\ell+2)'\,(\ell+1)'}(u) \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

and $H^{[\ell]}(u) = \operatorname{diag}\left[h_{\ell+1}(u), \dots, h_{(\ell+1)'}(u)\right]$. Furthermore, introduce the product of these matrices by

$$T^{[\ell]}(u) = F^{[\ell]}(u)\, H^{[\ell]}(u)\, E^{[\ell]}(u).$$

Accordingly, the entries of $T^{[\ell]}(u)$ will be denoted by $t_{ij}^{[\ell]}(u)$ with $\ell+1 \leqslant i, j \leqslant (\ell+1)'$.

The following properties of the Gauss decomposition observed in [18, Sec. 4] extend to the super case in the same form. We use the notation of Corollary 3.2.

Proposition 4.2. *The series $t_{ij}^{[\ell]}(u)$ coincides with the image of the generator series $t_{ij}(u)$ of the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m-2\ell})$ (for $\ell < m$) or the extended Yangian $\mathrm{X}(\mathfrak{o}_{N+2m-2\ell})$ (for $\ell \geqslant m$), under the embedding (3.14),*

$$t_{ij}^{[\ell]}(u) = \psi_\ell\big(t_{ij}(u)\big), \qquad \ell+1 \leqslant i, j \leqslant (\ell+1)'.$$

Moreover, the subalgebra $\mathrm{X}^{[\ell]}(\mathfrak{osp}_{N|2m-2\ell})$ generated by the coefficients of all series $t_{ij}^{[\ell]}(u)$ with $\ell+1 \leqslant i, j \leqslant (\ell+1)'$ is isomorphic to the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m-2\ell})$ (for $\ell < m$) or the extended Yangian $\mathrm{X}(\mathfrak{o}_{N+2m-2\ell})$ (for $\ell \geqslant m$). □

Introduce the coefficients of the series defined in (4.2), (4.3) and (4.4) by

$$e_{ij}(u)=\sum_{r=1}^{\infty}e_{ij}^{(r)}\,u^{-r},\qquad f_{ji}(u)=\sum_{r=1}^{\infty}f_{ji}^{(r)}\,u^{-r},\qquad h_i(u)=1+\sum_{r=1}^{\infty}h_i^{(r)}\,u^{-r}. \tag{4.6}$$

Furthermore, set

$$k_i(u)=h_i(u)^{-1}h_{i+1}(u),\qquad e_i(u)=e_{i\,i+1}(u),\qquad f_i(u)=f_{i+1\,i}(u), \tag{4.7}$$

with $i=1,\dots,m+n$ for type B and with $i=1,\dots,m+n-1$ for type D. In the latter case we also set

$$k_{m+n}(u)=h_{m+n-1}(u)^{-1}\,h_{m+n+1}(u) \tag{4.8}$$

and

$$e_{m+n}(u)=e_{m+n-1\,m+n+1}(u),\qquad f_{m+n}(u)=f_{m+n+1\,m+n-1}(u). \tag{4.9}$$

We will also use the coefficients of the series defined by

$$e_i(u)=\sum_{r=1}^{\infty}e_i^{(r)}u^{-r}\qquad\text{and}\qquad f_i(u)=\sum_{r=1}^{\infty}f_i^{(r)}u^{-r}. \tag{4.10}$$

The following is a super-version of [18, Lemma 4.3] (the case $m=0$ was covered therein); we will use a different argument.

Lemma 4.3. *Suppose that the indices i,j,k satisfy $\ell+1\leqslant i,j,k\leqslant(\ell+1)'$ and $k\neq j'$. Then the following relations hold in the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m})$,*

$$\Big[e_{\ell k}(u),t_{ij}^{[\ell]}(v)\Big]=\frac{1}{u-v}\,t_{ik}^{[\ell]}(v)\Big(e_{\ell j}(v)-e_{\ell j}(u)\Big)(-1)^{\bar{\imath}+\bar{k}+\bar{\imath}\bar{k}}, \tag{4.11}$$

$$\Big[f_{k\ell}(u),t_{ji}^{[\ell]}(v)\Big]=\frac{1}{u-v}\Big(f_{j\ell}(u)-f_{j\ell}(v)\Big)\,t_{ki}^{[\ell]}(v)(-1)^{\bar{\jmath}+\bar{k}+\bar{\jmath}\bar{k}}. \tag{4.12}$$

Proof. We will assume that $m\geqslant 1$ and start by verifying (4.11) for $\ell=1$. The defining relations (2.8) imply that under the given restrictions of the indices,

$$\Big[t_{1k}^{(1)},t_{ij}(v)\Big]=\delta_{ik}\,t_{1j}(v)(-1)^{\bar{\imath}+\bar{k}+\bar{\imath}\bar{k}},\qquad \Big[t_{1k}^{(1)},t_{11}(v)\Big]=t_{1k}(v),$$

and hence

$$\Big[t_{1k}^{(1)},t_{11}(v)^{-1}\Big]=-t_{11}(v)^{-1}t_{1k}(v)\,t_{11}(v)^{-1}.$$

By Proposition 4.2,

$$t_{ij}^{[1]}(v)=t_{ij}(v)-t_{i1}(v)\,t_{11}(v)^{-1}t_{1j}(v),$$

so that

$$\Big[t_{1k}^{(1)},t_{ij}^{[1]}(v)\Big]=t_{ik}^{[1]}(v)\,t_{11}(v)^{-1}t_{1j}(v)(-1)^{\bar{\imath}+\bar{k}+\bar{\imath}\bar{k}}.$$

On the other hand, Corollary 3.3 implies that

$$\Big[t_{11}(u),t_{ij}^{[1]}(v)\Big]=0.$$

By taking the super-commutator of the left hand side with $t_{1k}^{(1)}$, we get

$$\Big[t_{1k}(u), t_{ij}^{[1]}(v)\Big] = -\Big[t_{11}(u), t_{ik}^{[1]}(v)\, t_{11}(v)^{-1} t_{1j}(v)(-1)^{\bar{\imath}+\bar{k}+\bar{\imath}\bar{k}}\Big]$$

$$= \frac{1}{u-v}\, t_{ik}^{[1]}(v)\, t_{11}(v)^{-1}\Big(t_{11}(u)\, t_{1j}(v) - t_{11}(v)\, t_{1j}(u)\Big)(-1)^{\bar{\imath}+\bar{k}+\bar{\imath}\bar{k}}.$$

Therefore, multiplying from the left by $t_{11}(u)^{-1}$, we arrive at (4.11).

Relation (4.12) for $\ell = 1$ now follows from (4.11) by applying the anti-automorphism τ and using Lemma 4.1. A similar argument for $m = 0$ gives another proof of [18, Lemma 4.3]. The case of general values of ℓ follows by the application of the homomorphism ψ_ℓ and using (3.15) and Proposition 4.2. □

5 Multiplicative formula for $c(u)$

We will need a formula for the series $c(u)$ defined in (2.5) in terms of the Gaussian generators $h_i(u)$ with $i = 1, \dots, m+n+1$. To derive it, we first establish some relations between the series $h_i(u)$ and show that all their coefficients pairwise commute in $\mathrm{X}(\mathfrak{osp}_{N|2m})$.

Proposition 5.1. *The following relations hold in the extended Yangian* $\mathrm{X}(\mathfrak{osp}_{N|2m})$:

$$h_i(u)\, h_{i'}\Big(u - \frac{N}{2} + m - i + 1\Big) = h_{i+1}(u)\, h_{(i+1)'}\Big(u - \frac{N}{2} + m - i + 1\Big) \tag{5.1}$$

for $i = 1, \dots, m$, *and*

$$h_{m+j}(u)\, h_{(m+j)'}\Big(u - \frac{N}{2} + j + 1\Big) = h_{m+j+1}(u)\, h_{(m+j+1)'}\Big(u - \frac{N}{2} + j + 1\Big) \tag{5.2}$$

for $j = 1, \dots, n$ *if* $N = 2n+1$, *and for* $j = 1, \dots, n-1$ *if* $N = 2n$.

Proof. Denote the quasideterminant (3.5) by $s_{ij}(u)$ and assume that i and j run over the set $P = \{2, 3, \dots, 1'\}$. By applying (3.3), we get

$$s_{ij}(u) = \Big|T(u)_{\{1,i\},\{1,j\}}\Big|_{ij} = \Big|\Big(T(u)^{-1}\Big)_{PP}\Big|_{ji}^{-1}.$$

Hence, (2.5) and (3.2) imply that $s_{ij}(u)$ coincides with the (i,j) entry of the matrix

$$c(u+\kappa)\,\Big(T^t(u+\kappa)_{PP}\Big)^{-1}. \tag{5.3}$$

Write the matrix $T^t(v)_{PP}$ in the block form

$$T^t(v)_{PP} = \begin{bmatrix} A(v) & B(v) \\ C(v) & D(v) \end{bmatrix}$$

according to the partition $P = \{2, \dots, 2'\} \cup \{1'\}$ of the row and column numbers. In particular, we have $D(v) = t_{11}(v)$. Using the block multiplication of matrices (see e.g. [25, Lemma 1.11.1]), we find that for any $i, j \in \{2, \dots, 2'\}$ the (i, j) entry of the matrix $(T^t(v)_{PP})^{-1}$ is found by

$$T^t(v)_{ij} - T^t(v)_{i1'}\, t_{11}(v)^{-1}\, T^t(v)_{1'j}$$

which equals

$$t_{j'i'}(v)(-1)^{\bar{i}\bar{j}+\bar{j}}\,\theta_i\theta_j - t_{1i'}(v)\, t_{11}(v)^{-1}\, t_{j'1}(v)(-1)^{1+\bar{i}}\,\theta_i\theta_j.$$

Note that this coincides with the (i, j) entry of the matrix $\mathfrak{S}^t(v)$, where $\mathfrak{S}(v) = [\sigma_{ij}(v)]_{i,j=2}^{2'}$ with

$$\sigma_{ij}(v) = t_{ij}(v) - t_{1j}(v)\, t_{11}(v)^{-1}\, t_{i1}(v)\,(-1)^{(1+\bar{i})(1+\bar{j})}.$$

As a next step, verify the identity

$$\sigma_{ij}(v) = t_{11}(v)\, t_{11}(v+1)^{-1}\, s_{ij}(v+1). \tag{5.4}$$

Indeed, by the defining relations,

$$t_{11}(v+1)\, t_{1j}(v) = t_{1j}(v+1)\, t_{11}(v)$$

and so

$$t_{11}(v+1)\,\sigma_{ij}(v) = t_{11}(v+1)\, t_{ij}(v) - t_{1j}(v+1)\, t_{i1}(v)\,(-1)^{(1+\bar{i})(1+\bar{j})},$$

which equals

$$t_{11}(v)\, t_{ij}(v+1) - t_{i1}(v)\, t_{1j}(v+1)$$

by (2.8). Together with the relation

$$t_{i1}(v)\, t_{11}(v+1) = t_{11}(v)\, t_{i1}(v+1)$$

this yields (5.4). Thus, the matrix $S(u) = [s_{ij}(u)]_{i,j=2}^{2'}$ is related to $\mathfrak{S}(v)$ via

$$\mathfrak{S}(v) = h_1(v)\, h_1(v+1)^{-1}\, S(v+1).$$

Since $S(u)$ coincides with the submatrix of (5.3) corresponding to the rows and columns labelled by the set $\{2, \dots, 2'\}$, we derive the relation

$$S(u)^{-1} = c(u+\kappa)^{-1}\, h_1(u+\kappa)\, h_1(u+\kappa+1)^{-1}\, S^t(u+\kappa+1).$$

On the other hand, by Theorem 3.1, the matrix $S(u)$ satisfies the RTT relation (2.4) associated with the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m-2})$. Therefore, by (2.5) we have

$$S(u)^{-1} = c'(u+\kappa+1)\, S^t(u+\kappa+1),$$

where $c'(u)$ is the central series associated with $\mathrm{X}(\mathfrak{osp}_{N|2m-2})$. We thus get the recurrence relation

$$c(u) = \frac{h_1(u)}{h_1(u+1)}\, c'(u+1). \tag{5.5}$$

Furthermore, as we pointed out in (3.13), the series $c(u)$ and $c'(u)$ are found by

$$c(u) = h_1(u)\, h_{1'}(u-\kappa) \qquad \text{and} \qquad c'(u) = h_2(u)\, h_{2'}(u-\kappa-1),$$

where we have taken into account the consistency property of the Gauss decompositions of the matrices $T(u)$ and $S(u)$ as stated in Proposition 4.2. Hence, by relation (5.5) we get

$$h_1(u)\, h_{1'}(u-\kappa-1) = h_2(u)\, h_{2'}(u-\kappa-1). \tag{5.6}$$

The proof of relations (5.1) is completed by the application of the maps ψ_ℓ with the use of (3.15) and Proposition 4.2. Relations (5.2) can be deduced by the same argument, starting with the generator matrix $T(u)$ of the extended Yangian $\mathrm{X}(\mathfrak{o}_N)$, or just by applying the non-super version of (5.6) implicitly contained in [18, Sec. 5]; see also [19, Lemma 2.1]. □

Corollary 5.2. *The coefficients of all series $h_i(u)$ with $i = 1, 2, \dots, 1'$ pairwise commute in $\mathrm{X}(\mathfrak{osp}_{N|2m})$.*

Proof. Note that the subalgebra of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ generated by the coefficients of the series $t_{ij}(u)$ with $i, j \in \{1, \dots, m+n\}$ is isomorphic to the Yangian $\mathrm{Y}(\mathfrak{gl}_{n|m})$. The Gauss decomposition of the corresponding generator matrix $T(u)$ was used in [13] to get Drinfeld-type presentations of $\mathrm{Y}(\mathfrak{gl}_{n|m})$ and $\mathrm{Y}(\mathfrak{sl}_{n|m})$. By changing the parity assumptions of [13] to their opposites (see also [30]), we find that the coefficients of the series $h_i(u)$ with $i = 1, \dots, m+n$ generate a commutative subalgebra $\mathcal{A}$ of $\mathrm{Y}(\mathfrak{gl}_{n|m}) \subset \mathrm{X}(\mathfrak{osp}_{N|2m})$. The relations of Proposition 5.1 imply that the coefficients of the remaining series $h_i(u)$ with $i = m+n+1, \dots, 1'$ can be expressed in terms of the elements of the commutative subalgebra of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ generated by $\mathcal{A}$ and the coefficients of the central series $c(u)$. □

We can now derive a multiplicative formula for the series $c(u)$.

Theorem 5.3. *We have the relations in the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m})$:*

$$c(u) = \prod_{i=1}^{m} \frac{h_i(u+i-1)}{h_i(u+i)} \prod_{j=1}^{n} \frac{h_{m+j}(u+m-j+1)}{h_{m+j}(u+m-j)} \times h_{m+n+1}(u+m-n+1/2)\, h_{m+n+1}(u+m-n)$$

for $N = 2n+1$, and

$$c(u) = \prod_{i=1}^{m} \frac{h_i(u+i-1)}{h_i(u+i)} \prod_{j=1}^{n-1} \frac{h_{m+j}(u+m-j+1)}{h_{m+j}(u+m-j)} \times h_{m+n}(u+m-n+1)\, h_{m+n+1}(u+m-n+1)$$

for $N = 2n$.

Proof. The relations follow from the recurrence relation (5.5) and the formulas for the respective central series associated with the extended Yangians $\mathrm{X}(\mathfrak{o}_N)$; see [18, Thm. 5.8]. □

6 Drinfeld presentation of the extended Yangian

We will rely on the Drinfeld presentations of the Yangian $\mathrm{Y}(\mathfrak{gl}_{n|m})$ obtained in [13], and the extended Yangian $\mathrm{X}(\mathfrak{o}_N)$ obtained in [18], to derive the following Drinfeld-type presentation of the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$. We will use the series introduced in (4.7)–(4.9) along with

$$e_i^\circ(u)=\sum_{r=2}^{\infty}e_i^{(r)}u^{-r}\qquad\text{and}\qquad f_i^\circ(u)=\sum_{r=2}^{\infty}f_i^{(r)}u^{-r}.$$

Theorem 6.1. *The extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m})$ with $N\geqslant 3$ and $m\geqslant 1$ is generated by the coefficients of the series $h_i(u)$ with $i=1,\dots,m+n+1$, and the series $e_i(u)$ and $f_i(u)$ with $i=1,\dots,m+n$, subject only to the following relations, where the indices take all admissible values unless specified otherwise. We have*

$$\big[h_i(u),h_j(v)\big]=0, \tag{6.1}$$

$$\big[e_i(u),f_j(v)\big]=\delta_{ij}\,\frac{k_i(u)-k_i(v)}{u-v}\,(-1)^{\overline{i+1}}. \tag{6.2}$$

For $i\leqslant m+n$ and all j, and for $i=m+n+1$ and $j<m+n$ we have

$$\big[h_i(u),e_j(v)\big]=-(\varepsilon_i,\alpha_j)\,\frac{h_i(u)\big(e_j(u)-e_j(v)\big)}{u-v}, \tag{6.3}$$

$$\big[h_i(u),f_j(v)\big]=(\varepsilon_i,\alpha_j)\,\frac{\big(f_j(u)-f_j(v)\big)h_i(u)}{u-v}. \tag{6.4}$$

For $N=2n+1$ we have

$$\begin{aligned}\big[h_{m+n+1}(u),e_{m+n}(v)\big]&=\frac{1}{2(u-v)}h_{m+n+1}(u)\big(e_{m+n}(u)-e_{m+n}(v)\big)\\&\quad-\frac{1}{2(u-v-1)}\big(e_{m+n}(u-1)-e_{m+n}(v)\big)h_{m+n+1}(u)\end{aligned} \tag{6.5}$$

and

$$\begin{aligned}\big[h_{m+n+1}(u),e_{m+n}(v)\big]&=-\frac{1}{2(u-v)}\big(f_{m+n}(u)-f_{m+n}(v)\big)h_{m+n+1}(u)\\&\quad+\frac{1}{2(u-v-1)}h_{m+n+1}(u)\big(f_{m+n}(u-1)-f_{m+n}(v)\big),\end{aligned} \tag{6.6}$$

whereas for $N=2n$ we have

$$\big[h_{m+n+1}(u),e_{m+n}(v)\big]=\frac{h_{m+n+1}(u)\big(e_{m+n}(u)-e_{m+n}(v)\big)}{u-v} \tag{6.7}$$

and

$$\big[h_{m+n+1}(u),f_{m+n}(v)\big]=-\frac{\big(f_{m+n}(u)-f_{m+n}(v)\big)h_{m+n+1}(u)}{u-v}. \tag{6.8}$$

Moreover,

$$\Big[e_i(u), e_i(v)\Big] = (\alpha_i, \alpha_i)\, \frac{\big(e_i(u) - e_i(v)\big)^2}{2\,(u - v)}, \tag{6.9}$$

$$\Big[f_i(u), f_i(v)\Big] = -(\alpha_i, \alpha_i)\, \frac{\big(f_i(u) - f_i(v)\big)^2}{2\,(u - v)}, \tag{6.10}$$

and for $i < j$ we have

$$u\Big[e_i^{\circ}(u), e_j(v)\Big] - v\Big[e_i(u), e_j^{\circ}(v)\Big] = -(\alpha_i, \alpha_j)\, e_i(u)\, e_j(v), \tag{6.11}$$

$$u\Big[f_i^{\circ}(u), f_j(v)\Big] - v\Big[f_i(u), f_j^{\circ}(v)\Big] = (\alpha_i, \alpha_j)\, f_j(v)\, f_i(u). \tag{6.12}$$

We have the Serre relations

$$\sum_{\sigma\in\mathfrak{S}_k} \Big[e_i(u_{\sigma(1)}), \Big[e_i(u_{\sigma(2)}), \dots, \Big[e_i(u_{\sigma(k)}), e_j(v)\Big] \dots\Big]\Big] = 0,$$
$$\sum_{\sigma\in\mathfrak{S}_k} \Big[f_i(u_{\sigma(1)}), \Big[f_i(u_{\sigma(2)}), \dots, \Big[f_i(u_{\sigma(k)}), f_j(v)\Big] \dots\Big]\Big] = 0,$$

for $i \neq j$ with $k = 1 + |c_{ij}|$, and for $m \geqslant 2$ the super Serre relations

$$\Big[[e_{m-1}(u_1), e_m(u_2)], [e_m(u_3), e_{m+1}(u_4)]\Big] + \Big[[e_{m-1}(u_1), e_m(u_3)], [e_m(u_2), e_{m+1}(u_4)]\Big] = 0,$$

$$\Big[[f_{m-1}(u_1), f_m(u_2)], [f_m(u_3), f_{m+1}(u_4)]\Big] + \Big[[f_{m-1}(u_1), f_m(u_3)], [f_m(u_2), f_{m+1}(u_4)]\Big] = 0.$$

Proof. Relations (6.1) follow from Corollary 5.2. To verify the remaining relations, regard the Yangian $\mathrm{Y}(\mathfrak{gl}_{n|m})$ as the subalgebra of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ generated by the coefficients of the series $t_{ij}(u)$ with $1 \leqslant i, j \leqslant m + n$. In type D there is another embedding of the Yangian $\mathrm{Y}(\mathfrak{gl}_{n|m})$, as the subalgebra generated by the coefficients of the series $t_{ij}(u)$ with i, j running over the set $\{1, \dots, m + n - 1, (m + n)'\}$. For both types B and D we will also use the embedding of the extended Yangian $\mathrm{X}(\mathfrak{o}_N) \hookrightarrow \mathrm{X}(\mathfrak{osp}_{N|2m})$ provided by the homomorphism ψ_m; see Corollary 3.2.

Therefore, some sets of relations between the Gaussian generators follow from [13, Thm. 3] (via the change of parity); see also [30]. Furthermore, for the values of the indices $i \geqslant m+1$ the relations are implied by the Drinfeld presentation of $\mathrm{X}(\mathfrak{o}_N)$ given in [18][1]. Note also that most of the relations involving the series $f_i(u)$ follow from their counterparts involving $e_i(u)$ due to the symmetry provided by the anti-automorphism τ defined in (2.9).

Using these observations, we find that the only cases of (6.2) not covered by the embeddings and the symmetry are $i \leqslant m$ and $j = m + n$. In those cases, the relation follows from Corollary 3.3, except for $i = m$ and $n = 1$ where Lemma 4.3 should be invoked in the same way as in

[1] The counterpart of (6.6) therein should be corrected by swapping the factors, while the correct condition for the counterparts of (6.11) and (6.12) should be $i < j$.

[18, Prop. 5.11]. Apart from the Serre and super Serre relations, the remaining relations are verified in the same way as (6.2): if some cases are not covered by [13] and [18], then Corollary 3.3 or Lemma 4.3 should be used.

Turning to the Serre relations, note that the coefficients of the series $e_i(u)$ and $f_i(u)$ are stable under all automorphisms (2.6) and so they belong to the subalgebra $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ of $\mathrm{X}(\mathfrak{osp}_{N|2m})$. The relations will be verified in the proof of the Main Theorem in Section 7, where we will see that they are equivalent to the respective relations (1.7) and (1.8).

The above arguments show that there is a homomorphism

$$\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m}) \to \mathrm{X}(\mathfrak{osp}_{N|2m}), \tag{6.13}$$

where $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$ denotes the algebra with generators and relations as in the statement of the theorem and the homomorphism takes the generators $h_i^{(r)}$, $e_i^{(r)}$ and $f_i^{(r)}$ of $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$ to the elements of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ with the same name, where we use the expansions for $h_i(u), e_i(u)$ and $f_i(u)$ as in (4.6) and (4.10). We will show that this homomorphism is surjective and injective.

To prove the surjectivity, note the following consequences of (2.8):

$$\Big[t_{ij}(u), t^{(1)}_{j\, j+1}\Big] = t_{i\, j+1}(u)(-1)^{\bar{j}} \tag{6.14}$$

for $1 \leqslant i < j \leqslant m+n$ for $N = 2n+1$, and for $1 \leqslant i < j \leqslant m+n-1$ for $N = 2n$, while

$$\Big[t^{(1)}_{j\, j+1}, t_{i\,(j+1)'}(u)\Big] = t_{i j'}(u)(-1)^{\bar{j}}$$

for $1 \leqslant i \leqslant j \leqslant m+n$ for $N = 2n+1$, and for $1 \leqslant i \leqslant j \leqslant m+n-1$ for $N = 2n$. Moreover, if $N = 2n$, then we also have

$$\Big[t_{i\, m+n-1}(u), t^{(1)}_{m+n-1\,(m+n)'}\Big] = t_{i\,(m+n)'}(u)$$

for $1 \leqslant i \leqslant m+n-2$. These relations together with the Poincaré–Birkhoff–Witt theorem for the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$ imply that it is generated by the coefficients of the series $t_{ij}(u)$ with $1 \leqslant i, j \leqslant m+n+1$. Furthermore, it follows from the Gauss decomposition (4.1), that the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$ is generated by the coefficients of the series $h_i(u)$ for $i = 1, \dots, m+n+1$ together with $e_{ij}(u)$ and $f_{ji}(u)$ for $1 \leqslant i < j \leqslant m+n+1$.

By writing the above relations in terms of the Gaussian generators (cf. [18, Sec. 5]), we get

$$\Big[e_{ij}(u), e^{(1)}_{j\, j+1}\Big] = e_{i\, j+1}(u)(-1)^{\bar{j}} \tag{6.15}$$

with the same respective conditions on the indices as in (6.14), whereas

$$\Big[e^{(1)}_{j\, j+1}, e_{i\,(j+1)'}(u)\Big] = e_{i j'}(u)(-1)^{\bar{j}}, \tag{6.16}$$

for $1 \leqslant i < j \leqslant m+n$ for $N = 2n+1$, and for $1 \leqslant i < j \leqslant m+n-1$ for $N = 2n$. In both B and D types, we also have

$$\Big[e^{(1)}_{i\, i+1}, e_{i\,(i+1)'}(u)\Big] = e_{i i'}(u)(-1)^{\bar{i}} - e_{i\, i+1}(u)\, e_{i\,(i+1)'}(u) \tag{6.17}$$

for $i=1,\dots,m$, while

$$\big[e_{i\,m+n-1}(u),e^{(1)}_{m+n-1\,(m+n)'}\big]=e_{i\,(m+n)'}(u) \tag{6.18}$$

for $1\leqslant i\leqslant m+n-2$ in type D. These relations together with their counterparts for the coefficients of the series $f_{ji}(u)$, which are obtained by applying the anti-automorphism τ and Lemma 4.1, show that the coefficients of the series $h_i(u)$ for $i=1,\dots,m+n+1$ and $e_i(u)$, $f_i(u)$ for $i=1,\dots,m+n$ generate the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$ thus proving that the homomorphism (6.13) is surjective.

To prove the injectivity of the homomorphism (6.13), we will apply the argument originally used in [4] and then also in [13] and [18]. The first step is to observe that the set of monomials in the generators $h_i^{(r)}$ with $i=1,\dots,m+n+1$ and $r\geqslant 1$, and $e_{ij}^{(r)}$ and $f_{ji}^{(r)}$ with $r\geqslant 1$ and the conditions

$$\begin{aligned} &i<j\leqslant i' \qquad \text{for}\quad i=1,\dots,m \qquad \text{and}\\ &i<j<i' \qquad \text{for}\quad i=m+1,\dots,m+n, \end{aligned} \tag{6.19}$$

taken in some fixed order, is linearly independent in the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m})$. To see this, introduce an ascending filtration on $\mathrm{X}(\mathfrak{osp}_{N|2m})$ by setting $\deg t_{ij}^{(r)}=r-1$ for all $r\geqslant 1$. Denote by $\bar t_{ij}^{(r)}$ the image of $t_{ij}^{(r)}$ in the $(r-1)$-th component of the associated graded algebra $\mathrm{gr}\,\mathrm{X}(\mathfrak{osp}_{N|2m})$. Introduce the coefficients c_r of the series $c(u)$ defined in (2.5) by

$$c(u)=1+\sum_{r=1}^{\infty}c_r\,u^{-r}$$

and denote by $\bar c_r$ the image of c_r in the $(r-1)$-th component of $\mathrm{gr}\,\mathrm{X}(\mathfrak{osp}_{N|2m})$. As with the non-super case considered in [3, Cor. 3.10] (see also [14]), by the decomposition (2.7) and the Poincaré–Birkhoff–Witt theorem, the map

$$\bar t_{ij}^{(r)}\mapsto F_{ij}\,x^{r-1}(-1)^{\bar\imath}+\frac{1}{2}\,\delta_{ij}\,\zeta_r \tag{6.20}$$

defines an isomorphism

$$\mathrm{gr}\,\mathrm{X}(\mathfrak{osp}_{N|2m})\cong \mathrm{U}(\mathfrak{osp}_{N|2m}[x])\otimes\mathbb{C}[\zeta_1,\zeta_2,\dots],$$

where $\mathbb{C}[\zeta_1,\zeta_2,\dots]$ is the algebra of polynomials in variables ζ_r understood as the images of the respective central elements $\bar c_r$. Also, let $\bar e_{ij}^{(r)}$, $\bar f_{ji}^{(r)}$ and $\bar h_i^{(r)}$ be the respective images in the $(r-1)$-th component of $\mathrm{gr}\,\mathrm{X}(\mathfrak{osp}_{N|2m})$. Under the isomorphism (6.20), the elements $\bar e_{ij}^{(r)}$ and $\bar f_{ji}^{(r)}$ respectively correspond to $F_{ij}\,x^{r-1}(-1)^{\bar\imath}$ and $F_{ji}\,x^{r-1}(-1)^{\bar\jmath}$. Similarly, $\bar h_i^{(r)}$ corresponds to $F_{ii}\,x^{r-1}(-1)^{\bar\imath}+\zeta_r/2$ for $i=1,\dots,m+n$, whereas[2]

$$\bar h_{m+n+1}^{(r)}\mapsto\begin{cases}\zeta_r/2 & \text{for}\quad N=2n+1,\\ -F_{m+n\,m+n}\,x^{r-1}+\zeta_r/2 & \text{for}\quad N=2n,\end{cases}$$

[2]This corrects the formula for type D in [18, Sec. 5.5].

which follows from Theorem 5.3. Hence the claimed linear independence of ordered monomials in $\mathrm{X}(\mathfrak{osp}_{N|2m})$ is implied by the Poincaré–Birkhoff–Witt theorem for $\mathrm{U}(\mathfrak{osp}_{N|2m}[x])$.

As a next step, working with the algebra $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$, introduce its elements inductively, as the coefficients of the series $e_{ij}(u)$ for i and j satisfying (6.19), by using relations (6.15)–(6.18). Furthermore, the defining relations show that the map

$$\tau : e_i(u) \mapsto f_i(u), \qquad f_i(u) \mapsto e_i(u) \qquad \text{for} \quad i = 1, \dots, m+n, \tag{6.21}$$

and $\tau : h_i(u) \mapsto h_i(u)$ for $i = 1, \dots, m+n+1$ defines an anti-automorphism of the algebra $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$. Apply this map to the relations defining $e_{ij}(u)$ and use the first relation in (4.5) to get the definition of the coefficients of the series $f_{ji}(u)$ subject to the same conditions (6.19). The injectivity of the homomorphism (6.13) will be proved by showing that the algebra $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$ is spanned by monomials in $h_i^{(r)}$, $e_{ij}^{(r)}$ and $f_{ji}^{(r)}$ taken in some fixed order. Denote by $\widehat{\mathcal{E}}$, $\widehat{\mathcal{F}}$ and $\widehat{\mathcal{H}}$ the subalgebras of $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$ respectively generated by all elements of the form $e_i^{(r)}$, $f_i^{(r)}$ and $h_i^{(r)}$. Define an ascending filtration on $\widehat{\mathcal{E}}$ by setting $\deg e_i^{(r)} = r-1$. Denote by $\mathrm{gr}\,\widehat{\mathcal{E}}$ the corresponding graded algebra.

Let $\bar{e}_{ij}^{(r)}$ denote the image of the element $(-1)^{\bar{\imath}}\, e_{ij}^{(r)}$ in the $(r-1)$-th component of the graded algebra $\mathrm{gr}\,\widehat{\mathcal{E}}$. Extend the range of subscripts of $\bar{e}_{ij}^{(r)}$ to all values $1 \leqslant i < j \leqslant 1'$ by using the skew-symmetry conditions

$$\bar{e}_{ij}^{(r)} = -\bar{e}_{j'i'}^{(r)}\,(-1)^{\bar{\imath}\bar{\jmath}+\bar{\imath}}\,\theta_i\theta_j.$$

To establish the spanning property of the monomials in the $e_{ij}^{(r)}$ in the subalgebra $\widehat{\mathcal{E}}$, it will be enough to verify the relations

$$\begin{aligned}\big[\bar{e}_{ij}^{(r)}, \bar{e}_{kl}^{(s)}\big] = \delta_{kj}\,\bar{e}_{il}^{(r+s-1)} &- \delta_{il}\,\bar{e}_{kj}^{(r+s-1)}\,(-1)^{(\bar{\imath}+\bar{\jmath})(\bar{k}+\bar{l})}\\ &- \delta_{ki'}\,\bar{e}_{j'l}^{(r+s-1)}\,(-1)^{\bar{\imath}\bar{\jmath}+\bar{\imath}}\,\theta_i\theta_j + \delta_{j'l}\,\bar{e}_{ki'}^{(r+s-1)}\,(-1)^{\bar{\imath}\bar{k}+\bar{\jmath}\bar{k}+\bar{\imath}+\bar{\jmath}}\,\theta_i\theta_j.\end{aligned} \tag{6.22}$$

Note the relations for the elements $\bar{e}_{ij}^{(r)}$ implied by (6.15)–(6.18):

$$\bar{e}_{i\,j+1}^{(r)} = \big[\bar{e}_{ij}^{(r)}, \bar{e}_{j\,j+1}^{(1)}\big] \tag{6.23}$$

for $1 \leqslant i < j \leqslant m+n$ for $N = 2n+1$, and for $1 \leqslant i < j \leqslant m+n-1$ for $N = 2n$, whereas

$$\bar{e}_{ij'}^{(r)} = \big[\bar{e}_{j\,j+1}^{(1)}, \bar{e}_{i\,(j+1)'}^{(r)}\big] \tag{6.24}$$

for $1 \leqslant i < j \leqslant m+n$ for $N = 2n+1$, and for $1 \leqslant i < j \leqslant m+n-1$ for $N = 2n$. Relation (6.24) also holds for $i = j$ when $1 \leqslant i \leqslant m$, while

$$\bar{e}_{i\,(m+n)'}^{(r)} = \big[\bar{e}_{i\,m+n-1}^{(r)}, \bar{e}_{m+n-1\,(m+n)'}^{(1)}\big] \tag{6.25}$$

for $1 \leqslant i \leqslant m+n-2$ in type D.

Since the defining relations between the coefficients of the series $e_i(u)$ with $1 \leqslant i \leqslant m+n-1$ are the same as the respective relations in the Yangian $\mathrm{Y}(\mathfrak{gl}_{n|m})$, the argument in the proof of [13, Thm. 3] implies (6.22) for the case where all indices i, j, k, l do not exceed $m + n$. Similarly, if all the indices exceed m, then (6.22) follows from the corresponding relations obtained in the proof of [18, Thm. 5.14]. The remaining cases are verified by the same inductive arguments as in [13] and [18] with the use of relations (6.23)–(6.25).

By applying the anti-automorphism (6.21), we deduce from the spanning property of the ordered monomials in the elements $e_{ij}^{(r)}$, that the ordered monomials in the elements $f_{ji}^{(r)}$ span the subalgebra $\widehat{\mathcal{F}}$. It is clear that the ordered monomials in $h_i^{(r)}$ span $\widehat{\mathcal{H}}$. Furthermore, by the defining relations of $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$, the multiplication map

$$\widehat{\mathcal{F}} \otimes \widehat{\mathcal{H}} \otimes \widehat{\mathcal{E}} \to \widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$$

is surjective. Therefore, ordering the elements $h_i^{(r)}$, $e_{ij}^{(r)}$ and $f_{ji}^{(r)}$ in such a way that the elements of $\widehat{\mathcal{F}}$ precede the elements of $\widehat{\mathcal{H}}$, and the latter precede the elements of $\widehat{\mathcal{E}}$, we can conclude that the ordered monomials in these elements span $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$. This proves that (6.13) is an isomorphism. □

Let $\mathcal{E}$, $\mathcal{F}$ and $\mathcal{H}$ denote the subalgebras of $\mathrm{X}(\mathfrak{osp}_{N|2m})$ respectively generated by all elements of the form $e_i^{(r)}$, $f_i^{(r)}$ and $h_i^{(r)}$. Consider the generators $h_i^{(r)}$ with $i = 1, \dots, m+n+1$ and $r \geqslant 1$, and $e_{ij}^{(r)}$ and $f_{ji}^{(r)}$ with $r \geqslant 1$ and conditions (6.19). Suppose that the elements $h_i^{(r)}$, $e_{ij}^{(r)}$ and $f_{ji}^{(r)}$ are ordered in such a way that the elements of $\mathcal{F}$ precede the elements of $\mathcal{H}$, and the latter precede the elements of $\mathcal{E}$. The following is a version of the Poincaré–Birkhoff–Witt theorem for the orthosymplectic Yangian.

Corollary 6.2. *The set of all ordered monomials in the elements $h_i^{(r)}$, $e_{ij}^{(r)}$ and $f_{ji}^{(r)}$ with the respective conditions on the indices forms a basis of $\mathrm{X}(\mathfrak{osp}_{N|2m})$.* □

7 Proof of the Main Theorem

Using the series (4.7)–(4.9), introduce the elements $\kappa_{i\,r}$ and $\xi_{i\,r}^{\pm}$ of the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$ as the coefficients of the series

$$\kappa_i(u) = 1 + \sum_{r=0}^{\infty} \kappa_{i\,r}\, u^{-r-1} \qquad \text{and} \qquad \xi_i^{\pm}(u) = \sum_{r=0}^{\infty} \xi_{i\,r}^{\pm}\, u^{-r-1}$$

by setting

$$\begin{aligned}
\kappa_i(u) &= k_i\Big(u + (-1)^{\bar{i}}(m-i)/2\Big),\\
\xi_i^{+}(u) &= f_i\Big(u + (-1)^{\bar{i}}(m-i)/2\Big),\\
\xi_i^{-}(u) &= (-1)^{\bar{i}}\, e_i\Big(u + (-1)^{\bar{i}}(m-i)/2\Big),
\end{aligned}$$

for $i = 1, \dots, m+n$ in type B and for $i = 1, \dots, m+n-1$ in type D, together with

$$\kappa_{m+n}(u) = k_{m+n}\big(u - (n-1)/2\big),$$
$$\xi^{+}_{m+n}(u) = f_{m+n}\big(u - (n-1)/2\big),$$
$$\xi^{-}_{m+n}(u) = e_{m+n}\big(u - (n-1)/2\big),$$

in type D. Since the series $\kappa_i(u)$ and $\xi_i^{\pm}(u)$ are fixed by all automorphisms (2.6), the elements $\kappa_{i\,r}$ and $\xi^{\pm}_{ir}$ belong to the subalgebra $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ of the extended Yangian $\mathrm{X}(\mathfrak{osp}_{N|2m})$. Moreover, the decomposition (2.7) and Theorem 5.3 imply that these elements generate the Yangian $\mathrm{Y}(\mathfrak{osp}_{N|2m})$; cf. [18, Prop. 6.1].

Relations (1.1)–(1.6) of the Main Theorem are deduced from Theorem 6.1 in the same way as for the Yangians $\mathrm{Y}(\mathfrak{gl}_{n|m})$ in [13] and $\mathrm{Y}(\mathfrak{o}_N)$ in [18]. Now we prove relations (1.7) and (1.8) and show that they imply the Serre relations and super Serre relations in the algebra $\mathrm{X}(\mathfrak{osp}_{N|2m})$. We use the argument originated in the work of Levendorskiĭ [24, Lemma 1.4]. Relations (1.3) and (1.4) imply that, as in [24, Cor. 1.5], for a certain polynomial $\widetilde{\kappa}_{i\,r}$ in the variables $\kappa_{i\,p}$ with $p \leqslant r$ we have

$$\big[\widetilde{\kappa}_{i\,r}, \xi^{\pm}_{j\,s}\big] = \pm\,(\alpha_i, \alpha_j)\,\xi^{\pm}_{j\,r+s} + \text{ linear combination of } \xi^{\pm}_{j\,r+s-2p} \text{ with } 2p \leqslant r.$$

The same argument as in [24] shows that the Serre relations both in $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ and $\mathrm{X}(\mathfrak{osp}_{N|2m})$ are implied by the Serre relations in the Lie superalgebra $\mathfrak{osp}_{N|2m}$ via the embedding (2.10), which are particular cases of (1.7) with $r_1 = \dots = r_k = s = 0$.

It was already shown in [30, Remark 2.61] how the super Serre relations in Theorem 6.1 follow from relations (1.8) with the use of the polynomials $\widetilde{\kappa}_{i\,r}$. The same argument applies to prove that all relations of the form (1.8) are implied by their particular case with $r = s = 0$ which holds in $\mathfrak{osp}_{N|2m}$.

We thus have an epimorphism from the algebra $\widehat{\mathrm{Y}}(\mathfrak{osp}_{N|2m})$ defined in the Main Theorem to the Yangian $\mathrm{Y}(\mathfrak{osp}_{N|2m})$, which takes the generators $\kappa_{i\,r}$ and $\xi^{\pm}_{ir}$ of $\widehat{\mathrm{Y}}(\mathfrak{osp}_{N|2m})$ to the elements of $\mathrm{Y}(\mathfrak{osp}_{N|2m})$ denoted by the same symbols. On the other hand, use the isomorphism $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m}) \cong \mathrm{X}(\mathfrak{osp}_{N|2m})$ to define the automorphisms of the form (2.6) on the algebra $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$. The injectivity of the epimorphism $\widehat{\mathrm{Y}}(\mathfrak{osp}_{N|2m}) \to \mathrm{Y}(\mathfrak{osp}_{N|2m})$ follows from the observation that $\widehat{\mathrm{Y}}(\mathfrak{osp}_{N|2m})$ coincides with the subalgebra of $\widehat{\mathrm{X}}(\mathfrak{osp}_{N|2m})$ which consists of the elements stable under all these automorphisms.

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