From resolution of singularities to equisingularities

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Objects defined by polynomial, or analytic, equations can have singularities. For example, the cusp $y^2 - x^3 = 0$ has both partial derivatives vanish at 0. By a substitution $x = X$, $y = XY$, called a blow-up, the cusp is the image of a parabola, $X = Y^2$, which is smooth. We say the cusp has been desingularised by one blow-up. Similarly $y^2 = x^5$ is desingularised by two blow-ups.

Resolution of singularities amounts to desingularising an object by a succession of blow-ups.

When one wishes to classify singularities, one ought to begin with an equivalence relation. Consider, for example, the Whitney family: $W_t(x, y) = xy(x + y)(x - ty)$, for $t > 0$. Should W_t and $W_{t'}$ $(t,\, t' > 0)$ be declared equivalent? And in which sense?

The Equisingularity Problem is to search for a "nice and natural" equivalence relation between singularities.

In this lecture we briefly survey some recent developments on an intimate relationship between desingularisation and equisingularity. A substantial part of the lecture can be understood by students.