Assignment 1

Due Thursday, 10 March, at 17:00 in the assignment box for MATH3402. The box is located on Level 4 in the Mathematics (Priestley) building (67). It has a white cover and is in the bottom row.

Please use a cover sheet!

Clearly state your assumptions and conclusions, and justify all steps in your work.

Marks will be deducted for sloppy working.

Q1 Let (X, d) be a metric space and $x_0 \in X$. Is the following function continuous?

$$X \to \mathbb{R}, \qquad x \mapsto d(x, x_0)$$

Q2 The interval $[1, 2]$ is imbued with the metric induced from the Euclidean metric on $\mathbb{R}$, which makes it into a complete metric space.
Let $f\colon [1, 2] \to [1, 2]$ be defined by $f(x) = \dfrac{x+2}{x+1}$.

Show that f is well defined, and that it is a contraction of the interval $[1, 2]$. Determine the fixed point of f, as well as a contraction constant for f. Moreover, show that f is monotonically decreasing.

Q3 Let (X_n, d_n), $n \in \mathbb{N}$, be a sequence of metric spaces, and let $X = \prod_{n\in\mathbb{N}} X_n$ be the cartesian product of the X_n's. (The elements of X are of the form $x = (x_1, x_2, \ldots)$ with $x_n \in X_n$.)

For $x, y \in X$, define

$$d(x, y) = \sum_{n=1}^{\infty} \frac{1}{2^n} \, \frac{d_n(x_n, y_n)}{1 + d_n(x_n, y_n)}.$$

Show that (X, d) is a metric space.