

Assignment 4

Your solutions should be submitted by the beginning of the lecture on
Tuesday, 25 October 2011.

Please attach a cover sheet!

- Q1 (a) Let Y be a finite dimensional subspace of the infinite dimensional normed space X . Show that Y has empty interior in X .
- (b) Let P be the vector space of all real polynomials in one variable and let $\|\cdot\|$ be an arbitrary norm on P . Show that $(P, \|\cdot\|)$ is not a Banach space.
- Q2 Let X be a normed space. Show that the sequence $(x_n)_{n=1}^{\infty} \subseteq X$ converges weakly to $x_0 \in X$ if and only if the following two conditions are satisfied:
- (a) The set $\{\|x_n\| \mid n \in \mathbb{N}\} \subset \mathbb{R}$ is bounded, i.e. $\sup_{n \in \mathbb{N}} \|x_n\| < \infty$.
- (b) For a fixed set $A \subseteq X^*$ with the property that $\overline{\text{span } A} = X^*$, we have:

$$\lim_{n \rightarrow \infty} f(x_n - x_0) = 0$$

for all $f \in A$. Recall that $\text{span } A$ is the subspace of X^* consisting of all finite linear combinations of elements of A . The closure is taken with respect to the usual topology on X^* .

- Q3 Show that if $1 < p < \infty$, then $\overline{\text{span}\{e_n \mid n \in \mathbb{N}\}} = l_p$. Use this fact and Q2 to characterise weak convergence in l_p . (You don't need to prove that $(l_p)^* \cong l_q$ with $q = \frac{p}{p-1}$.)