LECTURE 3 (§12.3, §12.5)

$\boxed{i}$

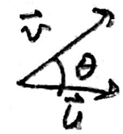
LAST TIME: VECTORS – ARROWS OR $\vec{v} = \langle v_1, \dots, v_n \rangle$

ARITHMETIC: ADDITION & SCALAR MULTIPLICATION

GEOMETRY: DOT PRODUCT $\vec{u} = \langle u_1, \dots, u_n \rangle$, $\vec{v} = \langle v_1, \dots, v_n \rangle$

$$\vec{u} \cdot \vec{v} = u_1 v_1 + \dots + u_n v_n$$

- $\vec{u} \cdot \vec{u} = |\vec{u}|^2$
- $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$



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STANDARD BASIS VECTORS:

$\mathbb{R}^2$: $\vec{i} = \langle 1, 0 \rangle$, $\vec{j} = \langle 0, 1 \rangle$ $\quad \vec{u} = \langle u_1, u_2 \rangle = u_1 \vec{i} + u_2 \vec{j}$

$\mathbb{R}^3$: $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$, $\vec{k} = \langle 0, 0, 1 \rangle$

$$\vec{u} = \langle u_1, u_2, u_3 \rangle = u_1 \vec{i} + u_2 \vec{j} + u_3 \vec{k}$$

$\mathbb{R}^n$: $\vec{e}_1 = \langle 1, 0, 0, \dots, 0 \rangle$, $\vec{e}_2 = \langle 0, 1, 0, \dots, 0 \rangle$, $\dots$, $\vec{e}_n = \langle 0, 0, \dots, 0, 1 \rangle$

$$\vec{u} = \langle u_1, \dots, u_n \rangle = u_1 \vec{e}_1 + \dots + u_n \vec{e}_n.$$

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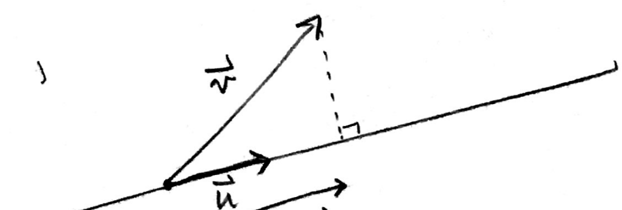
PROJECTIONS: $\vec{u}, \vec{v}$ VECTORS, $\vec{u} \neq \vec{0}$.



$\text{proj}_{\vec{u}} \vec{v}$ = PROJECTION OF $\vec{v}$ ALONG $\vec{u}$

$$= \underbrace{|\vec{v}| \cos\theta}_{} \underbrace{\frac{\vec{u}}{|\vec{u}|}} = \frac{|\vec{u}||\vec{v}| \cos\theta}{|\vec{u}|} \frac{\vec{u}}{|\vec{u}|} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2} \vec{u}$$

SIGNED LENGTH OF $\text{proj}_{\vec{u}} (\vec{v})$ $= \text{comp}_{\vec{u}} (\vec{v})$

unit vector (VECTOR OF MAG = 1) IN DIR. OF $\vec{u}$

NOTE: $\vec{u} = \langle u_1, u_2, u_3 \rangle$, THEN

$$\text{proj}_{\vec{i}} (\vec{u}) = u_1 \vec{i}, \quad \text{proj}_{\vec{j}} (\vec{u}) = u_2 \vec{j}, \quad \text{proj}_{\vec{k}} (\vec{u}) = u_3 \vec{k}$$

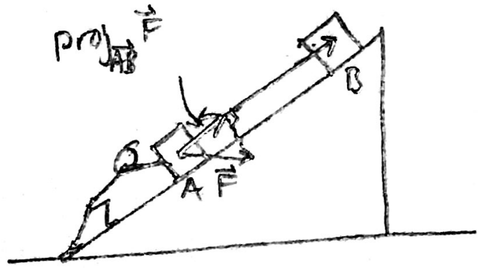
EX WORK = FORCE * DISTANCE

$$W = |\mathrm{proj}_{\vec{AB}} \vec{F}||\vec{AB}|$$

$$= \left|\frac{\vec{AB}\cdot\vec{F}}{|\vec{AB}|^2}\vec{AB}\right||\vec{AB}|$$

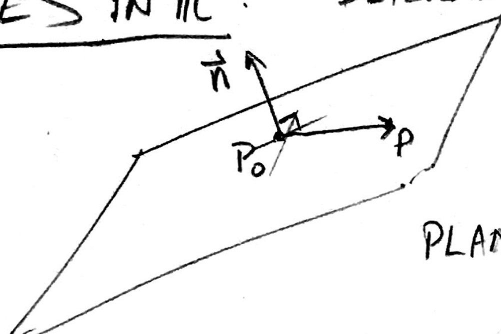
$$= \frac{|\vec{AB}\cdot\vec{F}|}{|\vec{AB}|^2}|\vec{AB}|^2$$

$$= \vec{AB}\cdot\vec{F} \quad (\text{ANGLE} \le \tfrac{\pi}{2}) \quad (\text{c.f. } \S 6.4)$$



PLANES IN $\mathbb{R}^3$:

DETERMINED BY
- POINT P_0 IN PLANE
- NORMAL VECTOR $\vec{n}$, ORTHOGONAL TO THE PLANE



A POINT P IN $\mathbb{R}^3$ IS IN THE PLANE IF AND ONLY IF $\vec{P_0P} \perp \vec{n}$ (ORTHOGONAL)

i

EQUATION? $P_0 = (x_0, y_0, z_0)$, $\vec{n} = \langle a, b, c\rangle$

Q WHEN IS $P = (x, y, z)$ IN THE PLANE DEFINED BY $P_0, \vec{n}$?

A IFF $\vec{P_0P}\cdot\vec{n} = 0$

IFF $\langle x - x_0, y - y_0, z - z_0\rangle \cdot \langle a, b, c\rangle = 0$

IFF $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

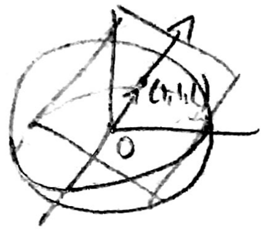
IFF $ax + by + cz \underbrace{-(ax_0 + by_0 + cz_0)}_{d} = 0$

IFF $\boxed{ax + by + cz + d = 0}$

EQN OF PLANE

ii

EX TANGENT PLANE TO SPHERE $x^2+y^2+z^2=3$ AT $(1,1,1)$.



RADIAL VECTOR TO $(1,1,1)$ IS ORTHOGONAL TO TANGENT PLANE, SO $\vec{n} = \langle 1,1,1\rangle$, $P_0 = (1,1,1)$

EQN:

$$\langle x-1, y-1, z-1\rangle \cdot \langle 1,1,1\rangle = 0$$

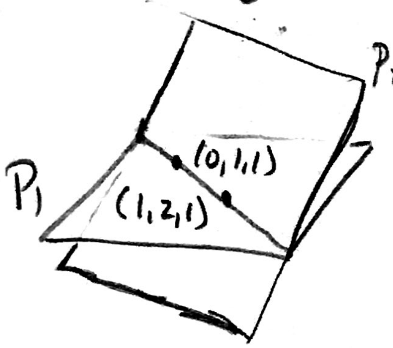
$$x-1+y-1+z-1=0$$

$$x+y+z=3$$

INTERSECTION OF PLANES:

EX P_1 = PLANE DEF'D BY $z-1=0$ $\qquad \vec{n}_1 = \langle 0,0,1\rangle$

P_2 = " " " $x-y+z=0$ $\qquad \vec{n}_2$



INTERSECTION IS A LINE. WHAT IS IT?

- FIND A COUPLE POINTS:

  E.G. $\overset{P}{(0,1,1)}$, $\overset{Q}{(1,2,1)}$

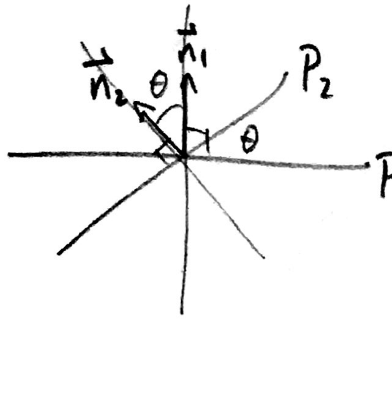
LINE IN DIRECTION $\overrightarrow{PQ}$, THROUGH $(0,1,1)$

$\boxed{i}$ VECTOR-VALUED FUNCTION DESCRIBING LINE

$$\vec{r}(t) = \overrightarrow{OP} + t\,\overrightarrow{PQ} = \langle 0,1,1\rangle + t\langle 1,1,0\rangle$$

$$= \langle t, 1+t, 1\rangle$$

ANGLE BETWEEN PLANES, θ, IS = ANGLE BETWEEN $\perp$ LINES



w/ $0 \le \theta \le \pi/2$, SO

$$\theta = \cos^{-1}\left(\frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}\right)$$

(WHY ABS. VALUE?