Last time: vector fields

Let C be the line segment from $(0, 0)$ to $(1, 2)$. Consider the vector field $\mathbf{F}(x, y) = \langle 1, 2y \rangle$.

What is $\int_C \mathbf{F} \cdot d\mathbf{r}$?

(a) 9

(b) 5

(c) 0

(d) 20

(e) I don't know what to do.

(If you're done, sketch the curve and the vector field, and check whether your answer is a reasonable one.)

Computing the integral of a vector field using the unit tangent vector

Consider the circle $C = \{x^2 + y^2 = 1\}$ oriented clockwise. Use the formula

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot \mathbf{T} ds$$

to find $\int_C \langle y, -x \rangle \cdot d\mathbf{r}$, *without* choosing a specific parametrization of C.

(a) π

(b) $-\pi$

(c) 2π

(d) -2π

(e) I don't know how.

If you're done, choose a parametrization and check your answer by computing the integral using the original definition.

Practice with the fundamental theorem of line integrals

Let C be a circle in $\mathbb{R}^2$ with centre P and radius r. Let $f(x, y) = 3x^2 + sin(x + y)$, and let $\mathbf{F} = \nabla f$.

What is $\int_C \mathbf{F} \cdot d\mathbf{r}$?

(a) Not enough information: I can't do it unless you tell me the starting and ending points of the path.

(b) Not enough information: I can't do it because you haven't told me the orientation of the circle.

(c) I think I can do it, but I need more time to compute this integral.

(d) It's zero.

Is the vector field conservative?

We're going to look at the vector field describing wind velocity.
Discuss with your neighbour: is this vector field conservative?
https://earth.nullschool.net/
(Remember the options below:)

(a) Yes, we think it is.

(b) No, we think it's not.

(c) We don't agree/we don't know.