Last time: directional derivative and gradient

Recall the definition of the gradient of a function $f : \mathbb{R}^n \to \mathbb{R}$:

$$\nabla f = \langle f_{x_1}, f_{x_2}, \dots, f_{x_n} \rangle.$$

Consider the function $f(x, yz) = x^2 + y^2 + z^2$. Find the equation of the plane through the point $(1, 1, 2)$ perpendicular to $\nabla f(1, 1, 2)$.

(a) $x + y + 2z = 6$
(b) $x + y + z = 4$
(c) $2x(x - 1) + 2y(y - 1) + 4x(z - 2) = 0$
(d) There is more than one such plane.
(e) I don't know.

Announcements

- Midterm 1 tomorrow evening (Tuesday).
  Bring your student ID.
- No lecture on Wednesday.
- Extra office hours:
  - Monday 2–3pm
  - Tuesday 11am–12:30pm
- Reduced office hours on Friday: 9–10am.

The tangent plane to a sphere

Let S be a sphere with centre $O = (0, 0, 0)$. Let P be a point on S.

Consider the following statement:

The tangent plane to S at P has normal vector $\overrightarrow{OP}$.

(a) This is always false.

(b) This depends on the specific sphere S and the point P.

(c) This is always true.

(d) I don't know.

Local maximum/minimum

Fix $f : \mathbb{R}^2 \to \mathbb{R}$, not necessarily differentiable; fix $(a, b) \in \mathbb{R}^2$.

- We say f has a local maximum at (a, b) if

$$f(a, b) \geq f(x, y) \text{ for all } (y, x) \text{ near } (a, b).$$

- We say f has a local minimum at (a, b) if

$$f(a, b) \leq f(x, y) \text{ for all } (y, x) \text{ near } (a, b).$$

Here "near (a, b)" means "for all (x, y) contained in a small disk of radius ϵ around the point (a, b)". (ϵ can be very small!)