

## Limits for functions of two (or more) variables

Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be a function such that

$$\lim_{x \rightarrow 0} f(x, 0) = \frac{1}{2}; \quad \lim_{y \rightarrow 0} f(0, y) = \frac{1}{2}.$$

What can we say about  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ ?

- (a) We don't have enough information to say anything.
- (b) We don't know if the limit exists, but if it does exist, it must be  $\frac{1}{2}$ .
- (c) The limit exists and is equal to  $\frac{1}{2}$ .
- (d) I don't understand the question.

Correct answer: (b)

## Make-up lecture on Wednesday's material

- Today, 3pm, right here in this room
- Lecture by Prof. Tolman; slides available from her website (link on Piazza course information page).
- Students from all sections welcome.
- 100% optional.

## Finding limits of continuous functions

Consider

$$f(x, y, z) = \frac{\sqrt{y}}{x^2 - y^2 + z^2}.$$

Find

$$\lim_{(x,y,z) \rightarrow (0,1,0)} f(x, y, z).$$

- (a) -1
- (b) 0
- (c) I've got  $\epsilon > 0$ , now I'm looking for  $\delta$ , and I need more time.
- (d) I don't know how to start.

Correct answer: (a). ((c) is not wrong, but it is not the most efficient approach.)

## Practice with partial derivatives

Let  $f(x, y) = \sin(3x + xy)$ . Calculate  $f_x(x, y)$ .

(a)  $\cos(3x + xy)$

(b)  $(3 + y) \sin(3x + xy)$

(c)  $(3 + y) \cos(3x + xy)$

(d)  $x \cos(3x + xy)$ .

Correct answer: (c)

## Practice with higher partial derivatives

Let  $f(x, y) = \sin(3x + xy)$ . Calculate  $f_{xy}(x, y)$ .

(a)  $-(3 + y)x \sin(3x + xy) + \cos(3x + xy)$

(b)  $(3 + y)x \sin(3x + xy) + \cos(3x + xy)$

(c)  $-(3 + y)x \sin(3 + xy)$

(d)  $(3 + y)x \sin(3 + xy) + x \cos(3x + xy)$

Correct answer: (a)