

**MATH 402 Homework 9**  
**Due Friday 16 November, 2018**

**Exercise 1.** Let  $P_n$  be a regular  $n$ -gon, and let  $G$  be its symmetry group.

- (a) Show that  $G$  is a *finite* symmetry group, by proving that there is an injective function from  $G$  into the set of permutations of  $n$  objects.
- (b) Show that  $G$  is the dihedral group  $D_n$  (i.e. it has exactly  $n$  rotations and  $n$  reflections). *Hints: it is enough to (a) prove that it is not cyclic and (b) prove that it has a rotation of order  $n$ , although you must explain why this is enough. Review the things we proved for the pentagon.*
- (c) Draw a figure whose symmetry group is cyclic, not dihedral.

**Exercise 2.** Let  $\ell$  and  $m$  be limiting parallel hyperbolic lines. Prove that they cannot have a common perpendicular. *Hint: consider angles of parallelism.*

**Exercise 3.** Let  $\ell$  and  $m$  be limiting parallel hyperbolic lines. Let  $r_\ell$  denote reflection across  $\ell$ , and let  $m' = r_\ell(m)$ . Prove that  $m'$  is also limiting parallel to  $\ell$ .

*Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.*