Examples of Chaotic behaviour

Memcapacitor model and its application in a chaotic oscillator*

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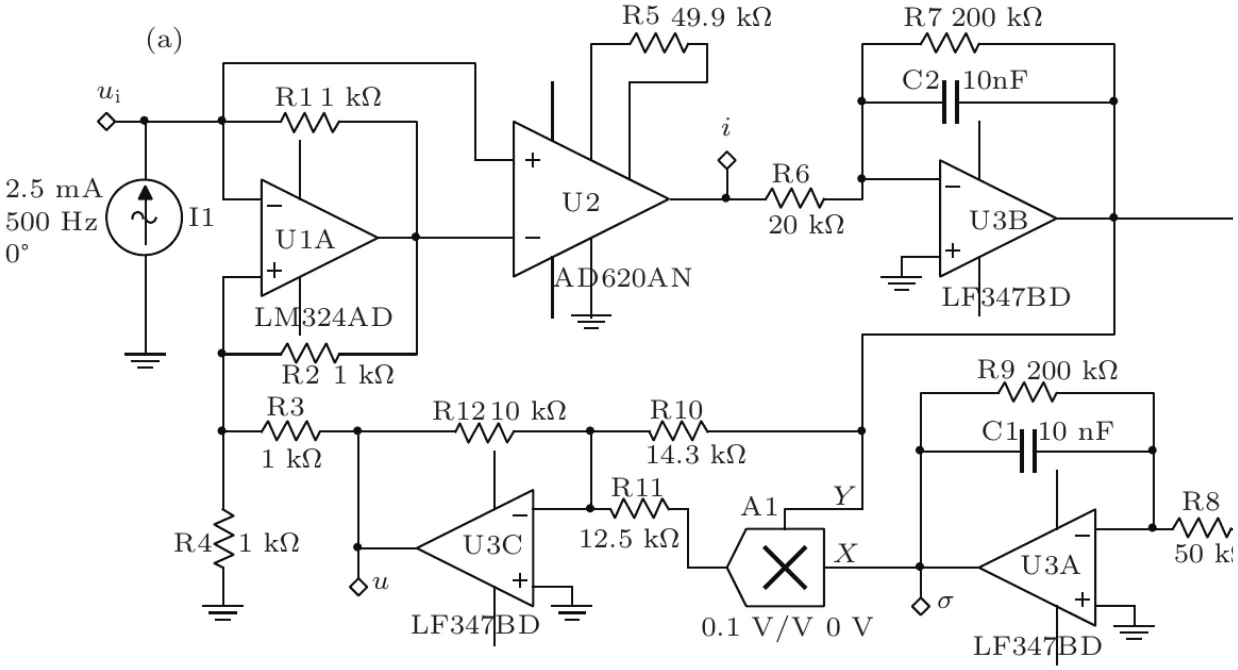

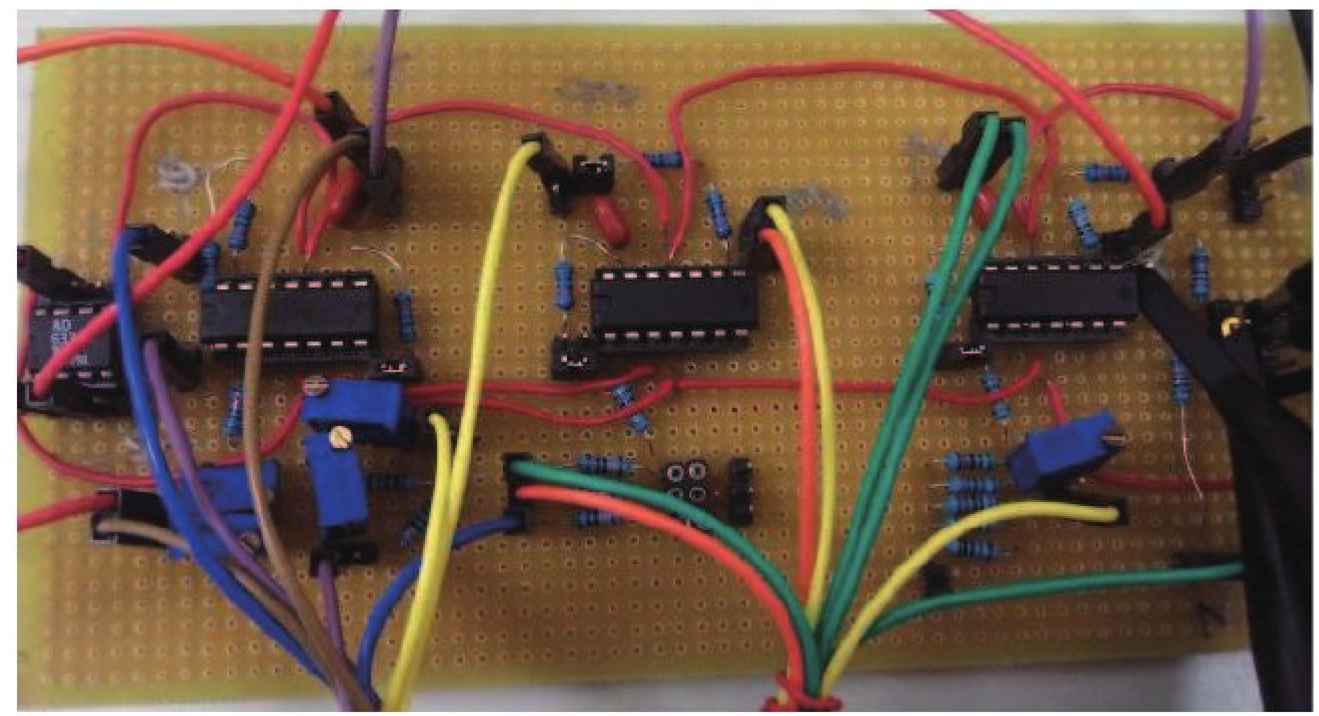

$$
\begin{cases}
\mathrm{d}x/\mathrm{d}\tau = m\left((e-1)\left(ax+bx^2\right)+y\right), \\
\mathrm{d}y/\mathrm{d}\tau = ax+bx^2-y+z, \\
\mathrm{d}z/\mathrm{d}\tau = -ky.
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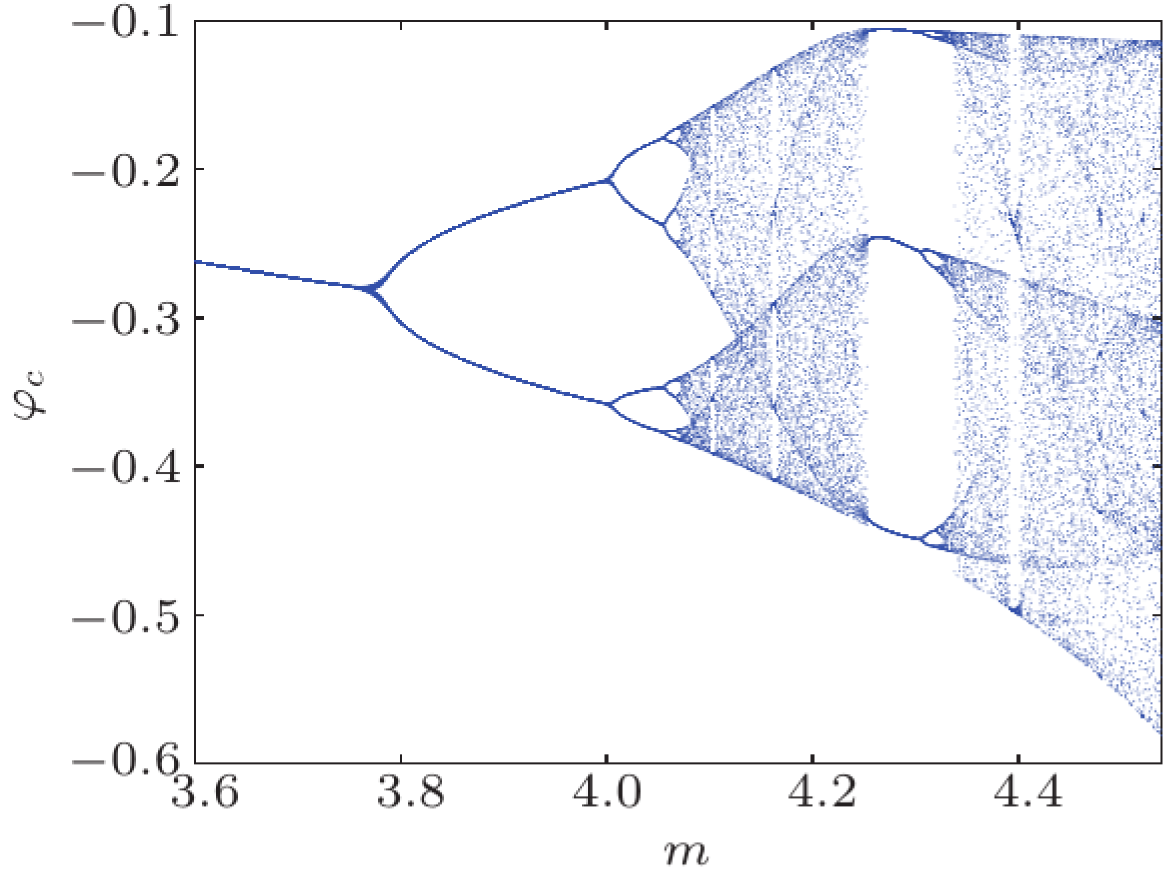

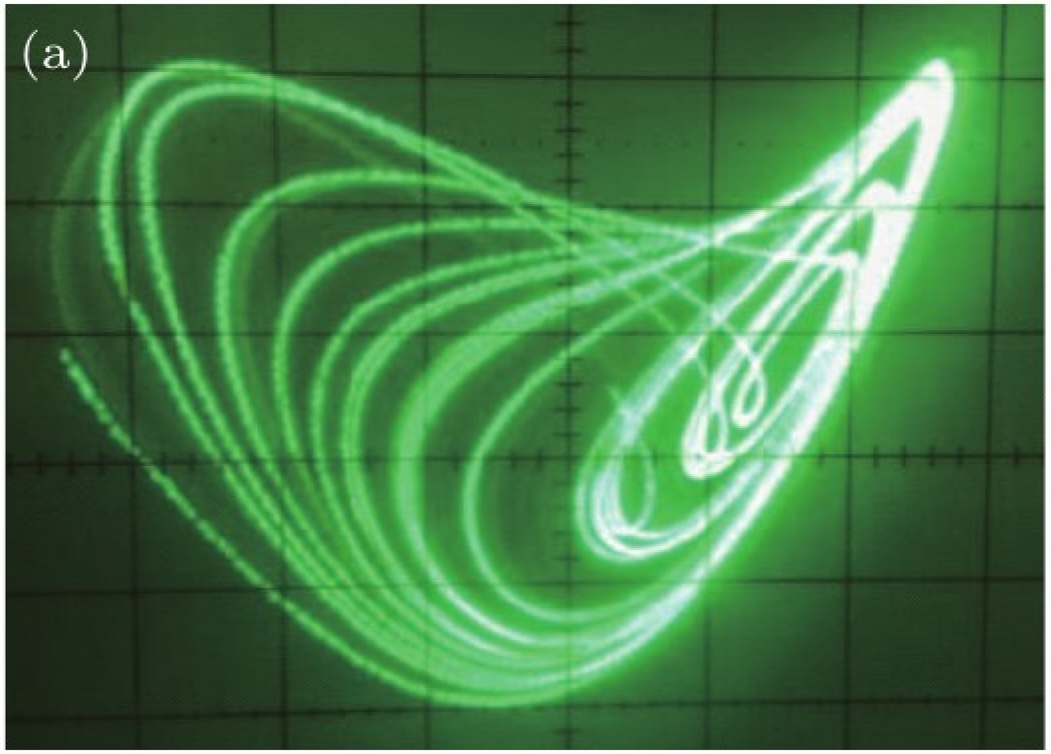

PHYSICAL REVIEW A VOLUME 43, NUMBER 5 1 MARCH 1991

Experimental investigation of the collision of Feigenbaum cascades in lasers

C. Lepers, J. Legrand, and P. Glorieux

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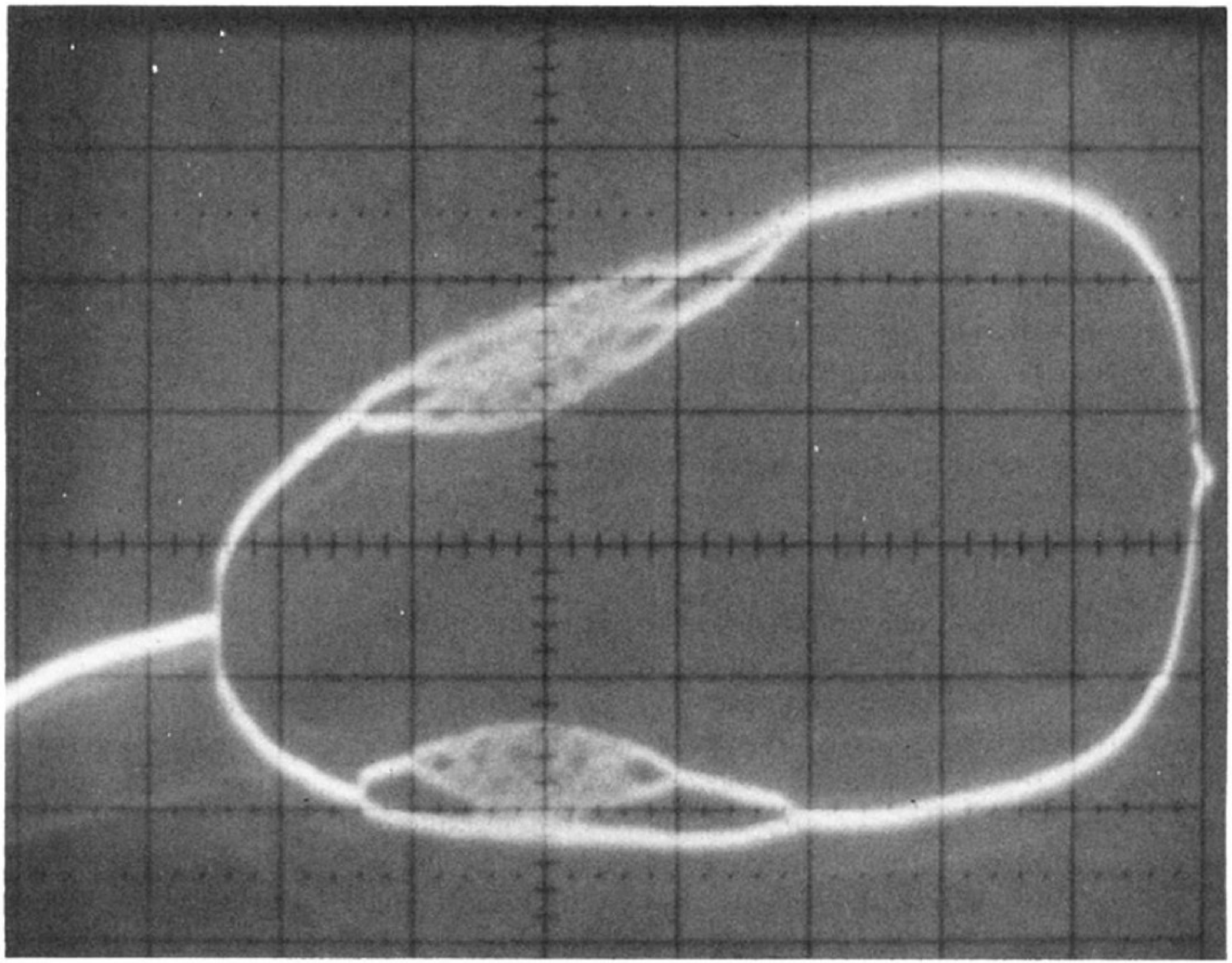
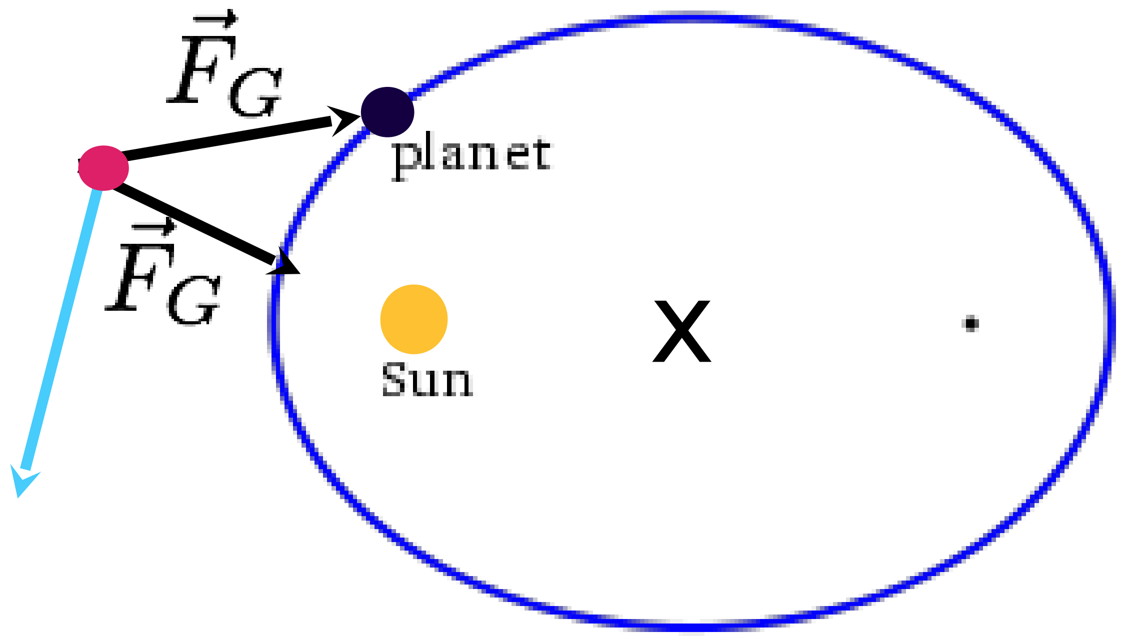


FIG. 1. Bifurcation diagram of the laser with modulated losses with the dc bias as control parameter ($60 < V_{dc} < 460$ V) and a fixed modulation ($V_m \simeq 3$ V).

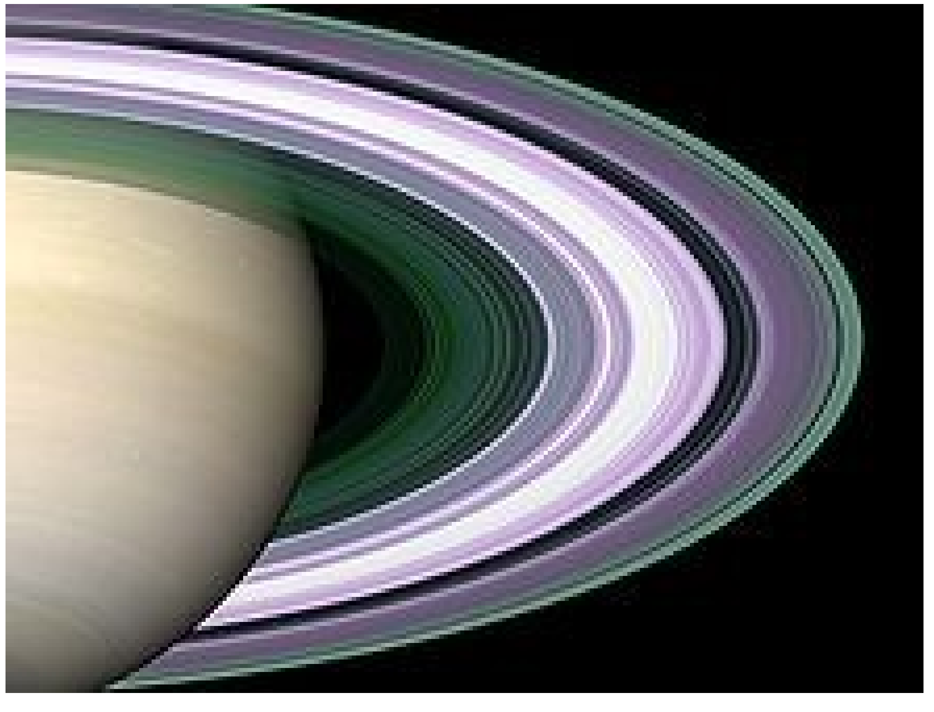
Chaos in the 3-body Problem
(Henri Poincaré,1854-1912)



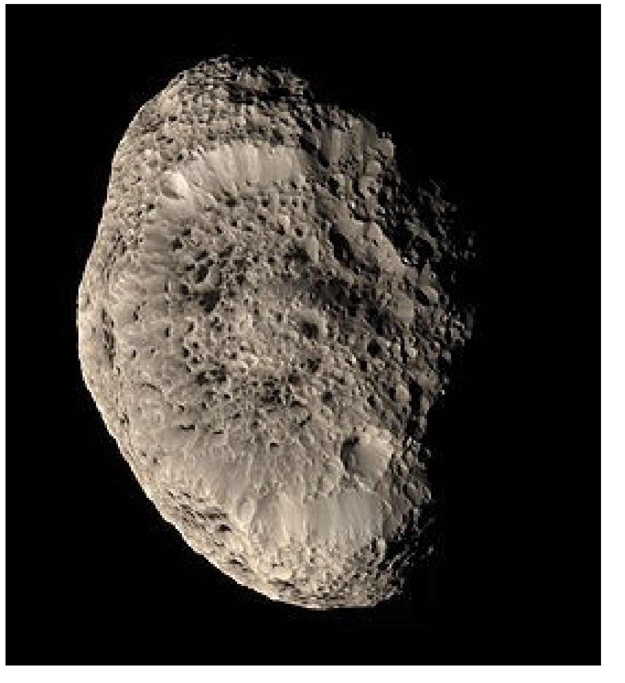
No closed curve

Chaos in Astronomy

Asteroids, e.g.,
522 Helga
(Lyap. time= 7kyr)



Saturn



Hyperion Moon

Chaos is typical in Astronomy

TABLE 1

Lyapunov Exponents and Times for the Solar System

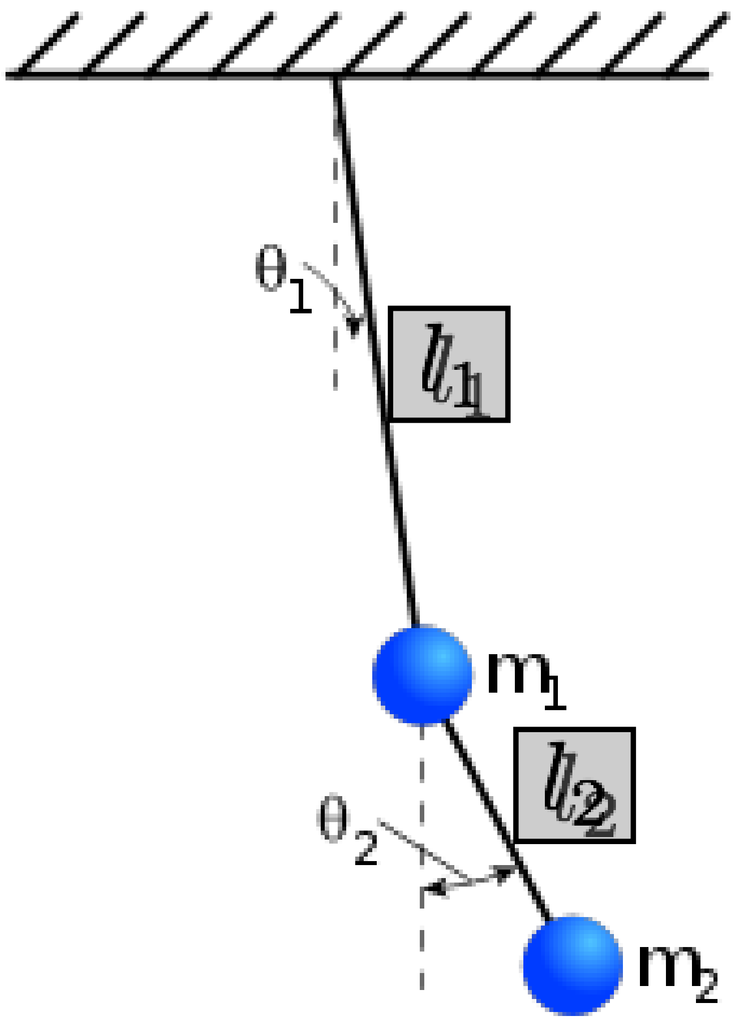
Planet	Lyapunov Exponent (yr^{-1})	Lyapunov Time (yr)
Mercury	7.32029×10^{-7}	1.36607×10^{6}
Venus	1.38561×10^{-7}	7.21703×10^{6}
Earth	2.07484×10^{-7}	4.81964×10^{6}
Mars	2.22353×10^{-7}	4.49736×10^{6}
Jupiter	1.19528×10^{-7}	8.36623×10^{6}
Saturn	1.56875×10^{-7}	6.37452×10^{6}
Uranus	1.33793×10^{-7}	7.47423×10^{6}
Neptune	1.49602×10^{-7}	6.68440×10^{6}

ON THE DYNAMICAL STABILITY OF THE SOLAR SYSTEM

Konstantin Batygin[1] and Gregory Laughlin[1,2]

The Astrophysical Journal, 683:1207–1216, 2008 August 20

Double Pendulum



$$L = K - V, \quad \frac{\partial L}{\partial \theta_i} = \frac{d}{dt}\frac{\partial L}{\partial \theta_i}$$

$$V = l_1(1 - cos\theta_1)m_1 g + [l_1(1 - \cos\theta_1) + l_2(1 - \cos\theta_2)]m_2 g$$

$$K = \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)$$

Non-linear Coupling