Recent Advances in Nonlinear PDE (UNE July 16-21, 2006): List of Talks

Herbert Amann Universität Zürich (Switzerland)

Parabolic equations on singular manifolds

We report on a rather simple and flexible approach to the study of linear and nonlinear parabolic equations on manifolds with corners, cusps, edges etc. The basic technique is illustrated by means of the heat equation on a manifold with a single cusp, with and without boundary conditions

Maria Athanassenas Monash University (Australia)

Recent developments in capillarity

Capillary surfaces (liquid-air or liquid-liquid interfaces) are mathematically modelled as surfaces of prescribed mean curvature. We will discuss two recent papers on compressible liquids, using classical non-linear, elliptic PDE techniques (with R. Finn), and measure-theoretical methods for energy minimisers (joint work with J. Clutterbuck)

Thomas Bartsch Universität Giessen (Germany)

Nodal solutions of semilinear boundary value problems

In this talk I report on the existence and on qualitative properties of nodal solutions of semi-linear boundary value problems. In particular the least energy nodal solution and nodal solutions with precisely two nodal domains will be discussed in detail.

Peter W. Bates Michigan State University (USA)

Invariant Manifolds of Spikes

Many singularly perturbed nonlinear elliptic equations have spike-like stationary solutions. These can be found through various methods, including Lyapunov-Schmidt schemes that, in the neighborhood of a proposed spike solution, decompose the operator equation into one that is restricted to a “normal subspace” and one in a “tangential subspace”. Here, these subspaces correspond to eigenstates of the operator, linearized at an approximate spike solution, and where “tangential” means “corresponding to eigenvalues near zero”, and “normal” means “complementary”. In this talk I will describe a more global decomposition in which the “tangential subspace” is replaced by a finite-dimensional manifold of spike-like states and this manifold is invariant with respect to the corresponding nonlinear parabolic equation and also normally hyperbolic. The stationary spike-like states lie on this manifold.

Jan Chabrowski University of Queensland (Australia)

The Neumann problem with critical Sobolev exponent

I will discus the existence results for the Neumann problem involving the critical Sobolev and Hardy exponents. The joint effect of the shape of the graphs of the coefficients and the shape of the boundary on the existence of solutions will be presented.

Nirmalendu Chaudhuri University of Wollongong (Australia)

On improved Hardy-Sobolev inequality and its application to partial differential equations with singular potential

It is well-known that the best constant in the classical Hardy-Sobolev inequality is never achieved in any domain, leaving room for an error estimate. In 1997 Brezis and Vasquez obtained an L^2-improved inequality. But their estimate is far from being optimal. In this talk we will discuss the optimal L^p-improved Hardy-Sobolev inequality in all dimensions. We will also discuss various applications of the improved inequality to semi-linear partial differential equations involving the critical inverse square potentials.

Florica Cirstea Australian National University

On some nonlinear elliptic problems with boundary blow-up

The study of elliptic problems with boundary blow-up has attracted considerable attention starting in 1916 with a work of Bieberbach. This interest has been reignited in the last decades from the need to undestand various models from Riemannian geometry, population dynamics and mathematical physics. In this talk we discuss the treatment of some nonlinear elliptic problems with boundary blow- up and their relationship with Karamata's theory of regular variation (and its extensions due to de Haan)

Massimo Grossi Universitá di Roma 1 (Italy)

Existence results for the Brezis-Nirenberg Problem in the supercritical case

Let us consider the equation $-\Delta u+a(|x|)u = u^p$ in the unit ball of $\mathbb{R}^n$, $n \geq 3$. We show, under suitable assumptions on a, the existence of positive solutions with zero Dirichlet boundary conditions if the exponent p is large enough. The proof of this result relies on the existence of a 'limit problem' as $p \to \infty$. This is a joint paper with Angela Pistoia.

Joseph Grotowski University of Queensland (Australia)

Two-dimensional harmonic map heat flowversus four-dimensional Yang-Mills heat flow

The two geometric flows in the title are the gradient flows associated to certain energy functionals. In the considered dimension, (i.e. dimension two for the harmonic map heat flow, dimension four for the Yang-Mills heat flow), the associated energy functional is (locally) conformally invariant, that is, the dimension is critical. In this talk we discuss similarities and differences between the flows in certain symmetric situations.

Adam Harris University of New England (Australia)

Embedding J-holomorphic cylinders asymptotic to a Reeb orbit of elliptic type

Pseudo-holomorphic mappings of a cylinder into the symplectisation of a contact manifold M give rise to interesting questions about asymptotic behaviour near periodic orbits of the Reeb flow. In particular, when the orbit is elliptic (i.e., the linearisation of the return map along the orbit has unimodular complex eigenvalues) we examine the extent to which the map may be represented holomorphically in a tubular neighbourhood.

Matthias Hieber TU Darmstadt (Germany)

Quasilinear systems with mixed boundary conditions

In this talk we investigate quasi-linear systems of reaction-diffusion equations with mixed Dirichlet-Neumann boundary conditions on non-smooth domains. Using techniques from maximal regularity and heat-kernel estimates we prove the existence of a unique solution to systems of this type. This is joint work with J. Rehberg (WIAS Berlin).

Danielle Hilhorst Université de Paris-Sud (France)

A reaction-diffusion system descriging tissue degradation by bacteria

We consider a model for the penetration of healthy tissue by bacteria from a burn wound. The mathematical formulation is given by a coupled system of two parabolic reaction-diffusion equations, together with homogeneous Neumann

boundary conditions and initial conditions. The unknown functions u_k and w_k are such that u_k corresponds to the concentration of degradative enzymes produced by the bacteria, and $1 - w_k$ corresponds to the volume fraction of healthy tissue. The key parameter $k > 0$ is typically very large and governs the degradation of ratio of the tissue. We describe the singular limit of the solution (u_k, w_k) as k tends to infinity and characterise the limit function pair (u_∞, w_∞) in terms of the weak solution of an auxiliary problem. Further results deal with the fast degradation rate limit of corresponding travelling wave solutions. This is joint work with John King and Matthias Röger.

Norimichi Hirano Yokohama National University (Japan)

Multiple existence of solutions for a non-homogeneous elliptic problem on $\mathbb{R}^N$

Let $N \geq 3$, $2^* = 2N/(N-2)$ and $p \in (2, 2^*)$. Our purpose in the paper is to consider the multiple existence of solutions of the problem

$$-\Delta u + u = |u|^{p-2}u + f, \qquad u \in H^1(\mathbb{R}^N),$$

where $f \in L^2(\mathbb{R}^N)$, $f \geq 0$ and $f \neq 0$.

Min-Chun Hong University of Queensland (Australia)

Stability of bi-harmonic maps into spheres

In this talk, we show that the equator map is a minimiser of the Hessian energy $H(u) = \int_\Omega |\nabla u|^2 \, dx$ in $H^2(\Omega; S^n)$ for $n \geq 10$ and is unstable for $5 \leq n \leq 9$.

Marek Izydorek Gdansk University of Technology (Poland)

Generalized heteroclinic solutions for a class of the second order Hamiltonian systems

We shall be concerned with the existence of heteroclinic orbits for the second order Hamiltonian system

$$\ddot{q} + V_q(t, q) = 0, \tag{HS}$$

where $q \in \mathbb{R}^n$ and $V \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$, $V \leq 0$. We will assume that V and a certain subset $M \subset \mathbb{R}^n$ satisfy the following conditions:

1. $\#M \geq 2$ and $\gamma := \inf\{|x - y| \colon x, y \in M, x \neq y\} > 0$,
2. there exists $0 < \varepsilon_0 \leq G$ such that for every $0 < \varepsilon \leq \varepsilon_0$ there is $\delta > 0$ such that for all $(t, x) \in \mathbb{R} \times \mathbb{R}^n$ if $d(x, M) \geq \varepsilon$ then $-V(t, x) > \delta$,
3. for every $x \in \mathbb{R}^n \setminus M$, $\underline{\lim}_{t \to \pm\infty} -V(t, x) = +\infty$,
4. for every $x \in M$, $\int_{-\infty}^{+\infty} -V(t, x) < \sqrt{2\alpha_0}$, where

$$\alpha_0 := \inf\{-V(t, x) \colon d(x, M) \geq \varepsilon_0\}.$$

Our result states that each point of M is joined with a certain other element of M by a solution of (HS). Since we should not expect that (HS) possesses a stationary solution the notion of a heteroclinic orbit is used in a generalized sense. Namely, $q \colon \mathbb{R} \to \mathbb{R}^n$ is *a generalized heteroclinic solution* of (HS) if there exist $x, y \in \mathbb{R}^n$, $x \neq y$ such that q joins x to y, (i.e. $\lim_{t \to -\infty} q(t) = x$, $\lim_{t \to +\infty} q(t) = y$).

Joanna Janczewska Gdansk University of Technology (Poland)

Homoclinic solutions for a class of Hamiltonian systems"

We study the existence of homoclinic orbits for the second order Hamiltonian system $\ddot{q} + V_q(t, q) = f(t)$, where $q \in \mathbb{R}^n$ and $V \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$, $V(t, q) = -K(t, q) + W(t, q)$ is T-periodic in t. A map K satisfies the "pinching" condition $b_1|q|^2 \leq K(t, q) \leq b_2|q|^2$, W is superlinear at the infinity and f is sufficiently small in $L^2(\mathbb{R}, \mathbb{R}^n)$. A homoclinic orbit is obtained as a limit of $2kT$-periodic solutions of a certain sequence of the second order differential equations.

James Kennedy University of Sydney (Austrialia)

Uniqueness in the Faber-Krahn inequality for Robin problems

We prove uniqueness in the Faber-Krahn inequality for the first eigenvalue of the Laplacian with Robin boundary conditions, asserting that amongst all sufficiently smooth domains of given volume, the ball is the unique minimiser for the first eigenvalue. The proof, which avoids the use of a symmetrisation of Schwarz, also works for Dirichlet boundary conditions. (Joint work with Daniel Daners)

Eun Heui Kim California State University Long Beach (USA)

Degenerate elliptic free boundary problems

In this talk, we discuss existence results and regularity results for free boundary value problems of a class of quasi-linear degenerate elliptic equations.

Hung-Ju Kuo National Chung-Hsing University (Taiwan)

On the fully non-linear elliptic difference equations

In this talk we present the following

1. Estimates of approximation solutions for stability
2. Applications to probability and tessellation of statistics
3. Discretisation for the partial differential equation $F(x, u, Du, D^2u) = 0$
4. The existence of solution of nonlinear difference equation $F[u] = F(x, u, Tu) = 0$.
5. Iteration method for computing approximation solution
6. Discrete regularity $C^{2+\alpha}$ for solution of $F[u](x) = F(x, L_1u(x), \ldots, L_ku(x)) = \psi(x)$

Yiming Long Nankai University (China)

Closed characteristics on convex hypersurfaces in $\mathbb{R}^6$

In this talk, I shall give a survey on recent results on closed characteristics on convex compact hyper-surfaces in $\mathbb{R}^{2n}$. Specially recently, jointly with W. Wang and X. Hu, we have proved that there exist at least three geometrically distinct closed characteristics on every smooth convex compact hyper-surface in $\mathbb{R}^6$.

Yichen Ma Xi'an Jiaotong University (China)

Local and parallel finite element algorithms based on two-grid discretization

Based on two-grid discretization we constract local and parallel finite element algorithms for three forms of the Navier-Stokes equations, which are penalty, primitive variable and stream function form. For every form the corresponding algorithm, the proof of convergence and error analysis and numerical results are given. The numerical simulations show these algorithms are reliable and efficient. (Joint work with Ma Feiyao, Zhang Zhipeng and Wang jian)

James McCoy University of Wollongong (Australia)

A Bernstein Property for a class of Monge-Ampère Equations

Inspired by recent work of Lin and Jia on a Bernstein property of affine maximal hypersurfaces of dimension $n = 2$ or $n = 3$, we generalise their result to n-dimensional hypersurfaces satisfying a related fourth order Monge-Ampère-type fully nonlinear PDE

Alan McIntosh Australian National University

Hardy spaces of differential forms on Riemannian manifolds

Let M be a complete Riemannian manifold. Assuming the doubling condition on the volume of balls, we define Hardy spaces H^p of differential forms on M and give various characterisations of them, including a molecular decomposition. As a consequence, we derive the H^p-boundedness for Riesz transforms on M, generalising previously known results. Further applications, in particular to functional calculus and Hodge decomposition, are given. This is joint work with Pascal Auscher and Emmanuel Russ.

Pengzi Miao Monash University (Australia)

A note on 3-dimensional positive Ricci curvature metrics with relatively large volume

Let g be a Riemannian metric with positive Ricci curvature on some closed manifold M^n. If the Ricci curvature of g is normalized to satisfy $Ric(g) \geq n-1$, then one knows by Bishop Volume Comparison theorem that the volume of (M^n, g) satisfies $Vol(g) \leq Vol(\mathbb{S}^n)$, where $\mathbb{S}^n$ is the standard n-dimensional unit sphere. On the other hand, it is a theorem of Colding that $Vol(g)$ is close to $Vol(\mathbb{S}^n)$ if and only if (M^n, g) is close to $\mathbb{S}^n$ in the Gromov-Hausdorff distance. In this talk, we consider a positive Ricci curvature metric g on the 3-dimensional sphere S^3 from a rather different point of view. We show that, if g satisfies $Ric(g) \geq 2$ and $Vol(g) \geq \frac{1}{2}Vol(\mathbb{S}^3)$, then the stereographic projection of (S^3, g) contains no closed minimal surfaces, hence generalizing a well known fact that the Euclidean space $\mathbb{R}^3$ has no closed minimal surfaces. Our consideration is motivated by the problem of existence of apparent horizons in general relativity.

Angela Pistoia Universitá di Roma 1 (Italy)

On the existence of sign changing solutions for the Bahri-Coron problem

Let Ω be a bounded smooth domain in $\mathbb{R}^N$, $N \geq 3$ and $p = \frac{N+2}{N-2}$. We are interested in existence and multiplicity of sign changing solutions to the slightly subcritical problem

$$(1) \qquad -\Delta u = |u|^{p-1-\epsilon}u \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega,$$

and to the Bahri-Coron's problem

$$(2) \qquad -\Delta u = |u|^{p-1}u \text{ in } \Omega_\epsilon,\ u = 0 \text{ on } \partial\Omega_\epsilon,$$

when $\Omega_\epsilon = \Omega \setminus B(0,\epsilon)$. In both cases ϵ is a small positive parameter. We prove that, problem (1) has at least N pairs of solutions which change sign exactly once. Moreover, the nodal surface of these solutions intersects the boundary of Ω, provided some suitable conditions are satisfied ([1]). When Ω is symmetric and contains the origin, we construct sign changing solutions to problems (1) ([3]) and (2) ([2]) with multiple blow up at the origin. These solutions have, as ϵ goes to zero, more and more annular-shaped nodal domains.

1. T. BARTSCH, A.M. MICHELETTI, A. PISTOIA, On the existence and the profile of nodal solutions of elliptic equations involving critical growth, *Calc. Var. Partial Differential Equations* (to appear).
2. M. MUSSO, A. PISTOIA, Sign changing solutions to Bahri-Coron's problem in pierced domains, (preprint).
3. A. PISTOIA, T. WETH, Sign changing bubble tower solutions in a slightly subcritical semilinear Dirichlet problem, *Ann. Inst. H. Poincaré Anal. Non Linéaire* (to appear).

Peter Polacik University of Minnesota (USA)

On a threshold behaviour in parabolic equations on $\mathbb{R}^N$

In a class of nonautonomous parabolic equations on the whole space, we consider solutions that separate two different kinds of asymptotic behavior (for example, convergence to two different constants or convergence to zero and blow up). We examine the uniqueness of such a separatrix on a ray of initial conditions and discuss the behavior of the corresponding solutions.

Pierre Portal Australian National University

Beyond Calderon-Zygmund theory in Bochner spaces

In the past few years many results on singular integral operators have been extended from the scalar-valued setting to the case of Bochner spaces $L_p(\mathbb{R}^n; X)$ where X is a UMD Banach space. The interest of such a generalization lies in its applications to nonlinear PDE and the insight it provides on some geometric properties of Banach spaces and on a probabilistic approach to harmonic analytic questions. This talk will present some of these results and, in particular, recent developments on operators which fall beyond the scope of Calderón-Zygmund theory.

Slawomir M. Rybicki Nicolaus Copernicus University (Poland)

Global bifurcations of solutions of elliptic systems

The aim of my talk is to present Rabinowitz alternative for systems of elliptic differential equations of the form

$$\begin{aligned} -\Delta u &= \nabla_u F(u,\lambda) && \text{in } \Omega, \\ u &= 0 && \text{on } \partial\Omega, \end{aligned}$$

where

1. $\Omega \subset R^N$ is an open, bounded subset of R^N, with boundary of the class C^{1-},
2. $F \in C^2(R^m \times R, R)$,
3. $dsF(x,\lambda) = \frac{\lambda}{2}\langle Ax, x\rangle + \eta(x,\lambda)$, where
   (a) A is a symmetric $(m \times m)$-matrix,
   (b) $\nabla_x \eta(0,\lambda) = 0$, for any $\lambda \in R$,
   (c) $\nabla_x^2 \eta(0,\lambda) = 0$, for any $\lambda \in R$,
4. there are $C > 0$ and $1 \le p < (N+2)(N-2)^{-1}$ such that for any $(x,\lambda) \in R^n \times R$ the following inequality holds true $|\nabla_x F(x,\lambda)| \le C\,(1 + |x|^p)$.

Gerd Schmalz University of New England (Australia)

CR-Manifolds, Differential Equations and Multicontact Structures

Cartan's method of moving frames has been successfully applied to the study of CR-manifolds, their mappings and invariants. For some types of CR-manifolds there is a close relation to the point-wise or contact geometry of differential equations. This can be used to find CR-manifolds with special symmetries. The recently introduced notion of multicontact structures provides a general framework comprising certain geometries of differential equations and CR-manifolds which in turn give examples with many symmetries.

Charles A. Stuart EPF Lausanne (Switzerland)

Hadamard differentiability and bifurcation

Abstract not available

Neil S. Trudinger Australian National University

Recent developmens in elliptic partial differential equations of Monge-Ampère type

I report on recent developments in the theory of Monge-Ampère equations, arising in optimal transportation and conformal geometry. The talk will mainly concern second derivative estimates and their application and is largely joint work with Xu-Jia Wang.

Xu-Jia Wang Australian National University

Fully non-linear partial differential equations and applications in geometry

We discuss recent progresses on the k-Hessian equation and its conformal counterpart. The k-Hessian equation is determined by the kth elementary symmetric polynomial of the eigenvalues of the Hessian matrix. It is the Laplace equation when $k = 1$ and a second order, fully nonlinear partial differential equation when $k \geq 2$. The k-Hessian equation is of divergent form. We will introduce various variational properties of the equation, in particular a Sobolev type inequality. In conformal geometry we are concerned with the existence of solutions to the corresponding conformal k-Hessian equation, namely the k-Yamabe problem. It is the classical Yamabe problem when $k = 1$. We show the existence of solutions if either $k \geq \frac{n}{2}$, or the equation is variational, which includes the cases when $k = 1, 2$ or when the manifold is locally conformal flat. The existence has also been established for equations determined by more general symmetric functions.

Juncheng Wei Chinese University of Hong Kong

On some supercritical problems

We consider two types of supercritical problems. The first one is the so-called Coron's problem: $\Delta u + u^p = 0, u > 0$ in $D = \Omega \backslash B_\delta(P)$, $u = 0$ on ∂D. We show that there exists resonant exponents $\frac{N+2}{N-2} < p_1 < p_2 < ... < p_j < ...$ such that for δ small, Coron's problem has a solution, provided $p > \frac{N+2}{N-2}$ and $p \neq p_j$. The second problem is nonlinear Schrodinger equation $\Delta u - V(x)u + u^p = 0, u > 0$in R^n, $\lim_{|x|\to+\infty} u(x) = 0$ We show that if $V(x) = o(\frac{1}{|x|^2})$, then for $p > \frac{N+1}{N-3}$, there is a continuum of positive solution. If $V(x)$ decays fast enough or $V(x)$ is symmetric in N directions, there is also a continuum of solutions when $\frac{N+2}{N-2} < p \leq \frac{N+1}{N-3}$.)

Anthony R. Weston Canisius College (USA)

Analysis of group structures induced by uniform homeomorphisms

Uniform homeomorphisms—that is to say, bijections which are uniformly continuous in both directions—induce group structures on complete normed vector spaces (Banach spaces) that "resemble" the additive structures of the underlying spaces.

Analysis of uniform Banach groups sheds new light on open questions about the classification of Banach spaces up to uniform homeomorphisms. For example, in this talk, we will detail a very general answer to a question recently highlighted by Yoav Benjamini and Joram Lindenstrauss in their monograph *Geometric Nonlinear Functional Analysis (Volume 1)*: Can a non-normable topological vector space be uniformly homeomorphic to a Banach space?

Our approach to this question, via uniform Banach groups, determines a new linearization procedure for a wide class of uniform homeomorphisms. Such techniques are rare because uniformly continuous maps do not have derivatives in general, rendering affine approximation difficult.

Kewei Zhang University of Sussex (UK)

The existence of Perona-Malik equation

We establish the existence of weak solutions for the one-dimensional version of the well-known Perona-Malik diffusion equation for edge enhancement in image processing by using the partial differential inclusion method.