WOMASY
Geometric and Harmonic Analysis meets PDE

University of Sydney

1st October 2014

Programm

Time	
10:00–10:05	**Welcome**
10:05–10:45	**Thierry Coulhon** (ANU) *Gaussian heat kernels estimates: from functions to forms*
10:50–11:10	**Morning Tea**
11:10–11:30	**Anthony Wong** (Macquarie University) *Besov Spaces Associated with Operators*
11:35–12:15	**Ting-Ying Chang** (University of Sydney) *Singular solutions for divergence-form elliptic equations involving regular variation theory*
12:15–13:45	**Lunch Break**
13:45–14:25	**Adam Sikora** (Macquarie University) *Harmonic Analysis for Grushin Type Operators*
14:30–16:10	**James McCoy** (University of Wollongong) *Curvature contraction of convex hypersurfaces by nonsmooth speeds*
16:10–15:25	**Afternoon Tea**
15:25–16:05	**Leo Tzou** (University of Sydney) *The Calderón Problem for Schrödinger Operators*
16:10–16:30	**Scott Parkins** (University of Wollongong) *The Generalised Polyharmonic Curve Flow of Closed Planar Curves*

For more detail see `http://www.maths.usyd.edu.au/u/PDESeminar/womasy/`

Abstracts of Talks

Singular solutions for divergence-form elliptic equations involving regular variation theory

Ting-Ying Chang (University of Sydney)

We generalise and sharpen several recent results by fully classifying the isolated singularities for nonlinear elliptic equations of the form

$$-\operatorname{div}\big(A(|x|)|\nabla u|^{p-2}\nabla u\big) + b_0(|x|)h(u) = 0 \quad \text{in } B_1(0)\setminus\{0\} \tag{1}$$

with $1 < p < \infty$. Here, $B_1(0)$ is the unit ball in $\mathbb{R}^N$ ($N \geq 2$). Let $A \in C^1(0,1]$ and $b_0 \in C(0,1]$ be positive functions, regularly varying at 0 with index ϑ and σ, whereas $h \in C[0,\infty)$ is positive non-decreasing on $(0,\infty)$ and regularly varying at ∞ with index q, where $q+1 > p > \vartheta - \sigma$.

For $p < N + \vartheta$, we show that 0 is a removable singularity for all positive solutions of (1) iff $b(x)h(\Phi) \notin L^1(B_1(0))$, where Φ denotes the "fundamental solution" of $-\operatorname{div}\big(A(|x|)|\nabla u|^{p-2}\nabla u\big) = \delta_0$ (the Dirac mass at 0) in $B_1(0)$. If, in turn, $b(x)h(\Phi) \in L^1(B_1(0))$, then we prove that (1) admits positive singular solutions with either a *weak* or a *strong* singularity at 0, finding a new explicit behaviour near zero in the latter case for the critical exponent. This is joint work with Florica Cîrstea.

Gaussian heat kernels estimates: from functions to forms

Thierry Coulhon (MSI, Australian National University)

Consider a non-compact Riemannian manifold where Gaussian upper estimates for the heat kernel on functions are satisfied. We give mild conditions on the Ricci curvature that ensure the validity of Gaussian estimates for the heat kernel on one-forms. This yields a new class of manifolds where the Riesz transform is bounded on L^p for all $p \in (1,+\infty)$. This is a joint work with Baptiste Devyver and Adam Sikora.

Curvature contraction of convex hypersurfaces by nonsmooth speeds

James McCoy (University of Wollongong)

We consider contraction of convex hypersurfaces by convex speeds, homogeneous of degree 1 in the principal curvatures, that are not necessarily smooth. We show how to approximate such a speed by a sequence of smooth speeds for which behaviour is well known. By obtaining speed and curvature pinching estimates for the flows by the approximating speeds, independent of the smoothing parameter, we may pass to the limit to deduce that the flow by the nonsmooth speed converges to a point in finite time that, under a suitable rescaling, is round in the C^2 sense, with the convergence being exponential.

This is joint work with Ben Andrews, Andrew Holder, Glen Wheeler, Valentina Wheeler and Graham Williams.

The Generalised Polyharmonic Curve Flow of Closed Planar Curves

Scott Parkins (University of Wollongong)

We consider a family of higher order curvature flows on closed planar curves (which includes the curve shortening flow and curve diffusion flow). We look at a natural energy called the "normalised oscillation of curvature" that measures how far a closed curve is from being an embedded circle (in an averaged L^2 sense not unlike the Willmore energy for closed surfaces). We then show that under any of these flows, closed curves suitably close to a circle (i.e. with a small normalised oscillation of curvature, as well as satisfying a suitable isoperimetric condition) exist for all time and converge exponentially fast to a round embedded circle.

Harmonic Analysis for Grushin Type Operators

Adam Sikora (Macquarie University)

We consider the classical Grushin operator $\partial_x^2 + x^2\partial_y^2$ and its natural generalization. For this class of degenerate elliptic operators we study some standard harmonic analysis type problems like heat kernel bounds, Poincaré inequalities, Riesz transform, convergence of eignefunction expansions, spectral multipliers or Bochner-Riesz type analysis.

The talk is based on a range of results obtained by Chen, Martini, Müller, Ouhabaz, Robinson and myself.

The Calderón Problem for Schrödinger Operators

Leo Tzou (University of Sydney)

The problem of determining the electrical conductivity of a body by making voltage and current measurements on the object's surface has various applications in fields such as oil exploration and early detection of malignant breast tumour. This classical problem posed by Calderón remained open until the late 1980s when it was finally solved in a breakthrough paper by Sylvester-Uhlmann.

In the recent years, geometry has played an important role in this problem. The unexpected connection of this subject to fields such as dynamical systems, symplectic geometry, and Riemannian geometry has led to some interesting progress. This talk will be an overview of some of the recent results and an outline of the techniques used to treat this problem.

The work described here is partially supported by NSF Grant No. DMS-0807502, Academy of Finland Fellowship 256378, Vetenskapsrådet 2012-3782

Besov Spaces Associated with Operators

Anthony Wong (Macquarie University)

In this talk we will study Besov spaces associated with operators. We let L be the generator of an analytic semigroup whose heat kernel satisfies an upper bound of Gaussian type acting on $L^2(X)$ where X is a (possibly non-doubling) space of polynomial upper bound on volume growth. We study a class of Besov spaces associated with the operator L so that when L is the Laplace operator or its square root acting on the Euclidean space $\mathbb{R}^n$, the Besov spaces are equivalent to the classical Besov spaces. Depending on the choice of L, the Besov spaces are natural settings for generic estimates for certain singular integral operators such as the fractional powers L^α. As an application, we study the decomposition of Besov spaces associated with Schrödinger operators with non-negative potentials satisfying reverse Hölder estimates.